Blackfolds

Meeting @Saclay, Nov 15, 2011

Niels Obers, NBI

0912.2352 (JHEP), 0910.1601 (JHEP), 0902.0427 (**PRL)** + 1106.4428 (JHEP)

(with R. Emparan, T. Harmark, V. Niarchos)

1110.4835 ((with J. Armas, J. Camps, T. Harmark)

1012.5081 (PRD) (with J. Armas)

1012.1494 (JHEP), 1101,1297 (NPB), 11xx.xxxx (to appear)

with G. Grignani, T. Harmark, A. Marini, M. Orselli

0708.2181 (JHEP) (with R. Emparan, T. Harmark, V. Niarchos, M.J. Rodriguez)

Intro +overview

- motivation to study black holes ? (topic of this conference…) :

will review aspects of blackfold (BF) method developed in the last five years
- effective theory of black brane dynamics

(reviews see e.g. Emparan,Harmark,Niarchos,NO, Emparan, NO)

dynamics of BHs in higher dimensions (especially D >5) too complex to be described by conventional methods

BF is an effective description in terms of a fluid living on a dynamical worldvolume
validity: horizon exhibits two (or more) widely separated length scales
(not possible in 4D pure Einstein gravity)

but: can infer properties that go beyond by extrapolation
e.g. instabilities, horizon topology changing mergers,
hints for new (extremal) exact solutions, checks on numerics

when applied to D-branes in ST: description of thermal D-branes that goes beyond the DBI description

Relvance of BF method

- new stationary BH solutions:

approximate analytic construction of BH metrics in higher D gravity/ supergravities (cf. String Theory)

- possible horizon topologies, thermodynamics, phase structure, …
- new non-extremal and extremal BH solutions
- useful for insights/checks on exact analytic/numeric solutions

(see talk by Maria Jose Rodriguez)

- BH instabilities and response coefficients:

understand GL instabilities in long wavelength regime, dispersion relation, elastic (in) stabilities, new long wavelength response coefficients for BHs, Young modulus (hydro + material science)

(see talk by Joan Camps)

- Thermal probe branes/strings:

new method to probe finite T backgrounds with probes that are in thermal equilibrium with the background (e.g. hot flat space, BHs)

(see talk by Troels Harmark)

- AdS/CFT:

many potential applications
(new black objects in AdS, connection with fluid/gravity, thermal probes thermal giant gravitons, BHs on branes, …)

+ interrelations between the four items above !

Plan

- Introduction : Separation of scales in higher-dimensional black holes
- Effective worldvolume theory: Blackfold (BF) approach
- Blackfolds in string theory and supergravity
- (An)isotropic effective fluids and new (non)-extermal ST BHs
- Discussion and outlook

- BF as a tool for thermal probe branes/strings (hot bion) (TH)
- Instabilities, correlated stability conjecture, viscosity and fine structure of BFs (internal spin, Young modulus) (JC)

Higher-dimensional gravity/ST

- In this talk: restrict mostly to
  asymptotically flat BH solutions of pure gravity and supergravity
  (including those relevant to ST)
  but:
  - interesting parallels with BHs in KK spaces
  - techniques are readily generalized to AdS/dS space

Dynamics of BHs in D > 4 much richer than four dimensions

- D=4: black hole uniqueness

- D=5: - 4D inspired techniques successful
  e.g. pure gravity:
  assuming 2 axial Killing vector fields $\longrightarrow$ integrability (inv. scattering)
  (see talk by MJR)

D > 5: only few known exact solutions
- full dynamics too complex to be captured by conventional approaches

$\Rightarrow$ blackfold approach

Higher D black holes organized according to scales

- dynamics of higher-dimensional black holes naturally organized in relative value of scales

$\ell_J \lesssim \ell_M$

• single length scale: Kerr BH behavior

$\ell_J \approx \ell_M$

• regime of mergers and connections between phases when two horizon scales meet r_0 » R

- not accessible to effective methods; requires extrapolation or numerics

e.g. Dias, Figueras, Monteiro, Santos, Emparan

$\ell_J \gg \ell_M$

• separation of scales allows effective description of long-wavelength description physics

Based on idea that when $\ell_J \gg \ell_M$ black hole is locally a flat (possibly boosted) black brane (cf. known examples)

Effective theory describes how to bend black brane wv in background spacetime

Blackfold = **Black** brane whose worldvolume extends along a curved submani**fold** of background spacetime

- to leading order in ℓ_M/ℓ_J `test' blackfold (neglect backreaction)

Blackfolds: A new framework

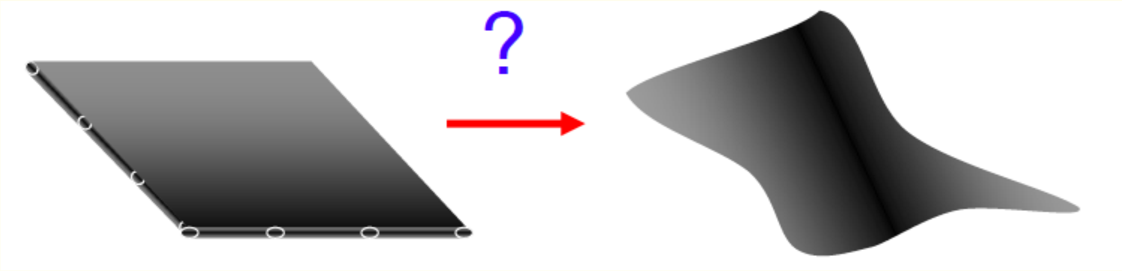
- identify why there is novel dynamics
  - different length scales along the horizon

  find new organizing framework
  - organize black holes according to scale hierachy

  develop new approaches to deal with it – new tools
  - effective theory at long wavelengths

→ effective fluid living on a dynamical worldvolume

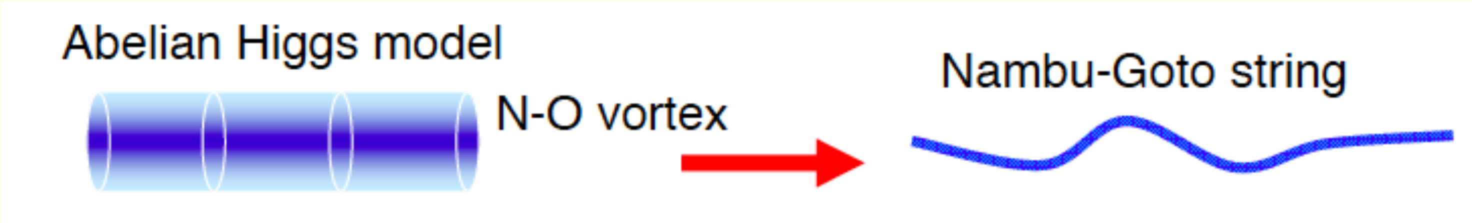
based on bending/vibrating of (flat) black branes



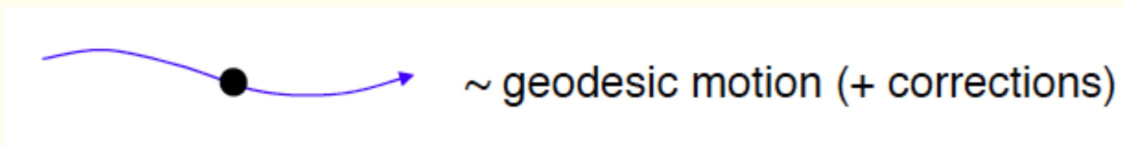
⇨ very much like other solitonic objects
(with in addition: worldvolume thermodynamics)

Similar systems

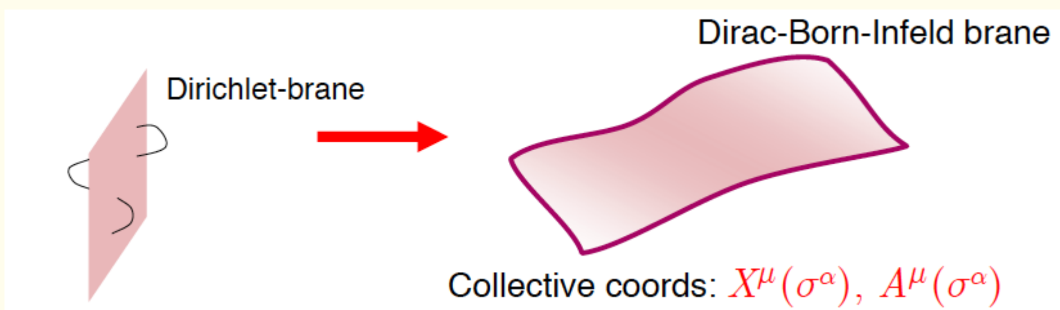
- Nielsen-Olesen vortices and NG strings



- small black holes in point-particle limit



- open strings and DBI action (effective theory for massles modes)

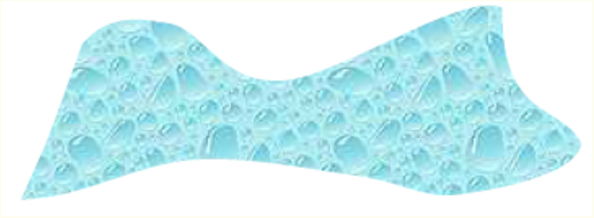


- fluid/gravity duality
(long-wavelength fluctuations of worldvolume = fluid dynamics)

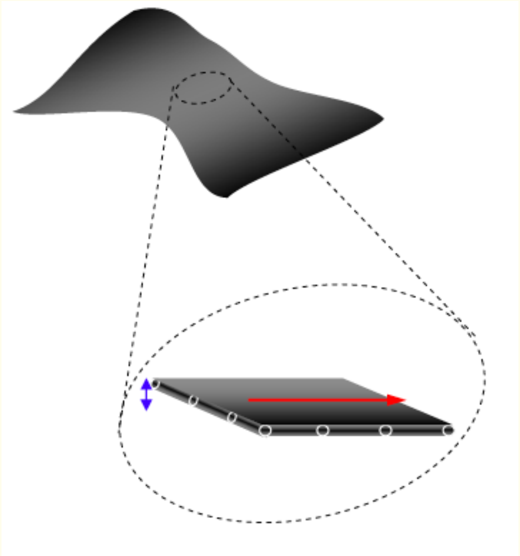
Main ingredients + results

- Ingredients:
  - classical brane dynamics (Carter)
  - long wavelengths: dynamics of fluid that lives on dynamical worldvolume
  - black branes correspond to specific type of fluid

    to leading order: perfect fluid



black hole looks locally like a flat black brane



for charged black branes of sugra:
novel type of (an)isotropic charged fluids

BF equations

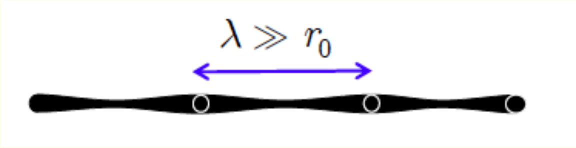
blackfold equations

Emparan,Harmark,Niarchos,NO

(liquid)

intrinsic (Euler equations of fluid
+ charge conservation)

fluid excitations (+ charge waves)



(solid)

extrinsic (generalized geodesic eqn. for
brane embedding)

elastic deformations

- gives novel stationary black holes (metric/thermo) + allows study of time evolution
- generalizes (for charged branes) DBI/NG to non-extremal solns. (thermal)
- possible in principle to incorporate higher-derivative corrections
  (self-gravitation + internal structure/multipole)

cf. closely related precedents of mappings black holes to fluid dynamics
- membrane paradigm
- fluid/AdS-gravity correspondence

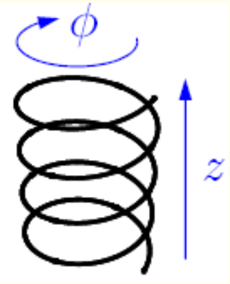
Neutral blackfolds

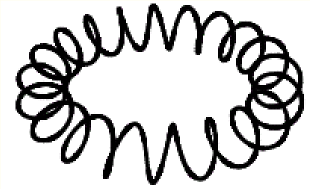
Emparan,Harmark,Niarchos,NO

First applied to neutral black branes of higher dim gravity:

Quick overview of results:

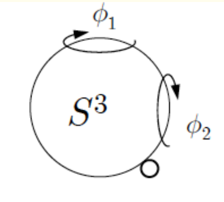
-new helical black strings and rings



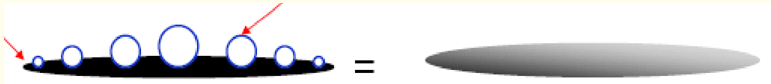


- odd-branes wrapped on odd-spheres
  (generalizes 5D black ring)

$$(\Pi_{p_a=\mathrm{odd}} S^{p_a}) \times s^{n+1}$$

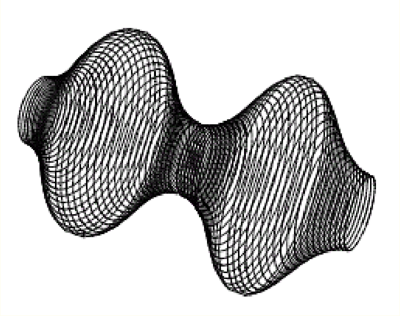


- even-branes wrapped on even-balls
  correctly reproduce MP BHs in
  ultraspinning (pancaked) limit

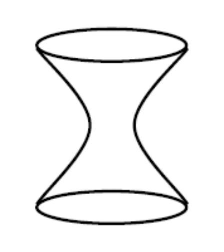


- non-uniform black cylinders

$$(\mathbb{R} \times S^1) \times s^{n+1}$$



- static minimal blackfolds
  (non-compact)



Blackfold Bestiary

blackfold construction shows existence of new types of asymptotically flat stationary black holes in higher dimensions

$D=4$	$D=5$	$D=6$	$D=7$	$D=8$	$D=9$	
S^2	S^3	S^4	S^5	S^6	S^7	Kerr, MP BH
		$B_2 \otimes s^2$	$B_2 \otimes s^3$	$B_2 \otimes s^4$ $B_4 \otimes s^2$	$B_2 \otimes s^5$ $B_4 \otimes s^3$	ultraspinning MP BH
	$S^1 \times s^2$	$S^1 \times s^3$	$S^1 \times s^4$	$S^1 \times s^5$	$S^1 \times s^6$	black ring
		$\mathbb{T}^2 \times s^2$	$\mathbb{T}^2 \times s^3$	$\mathbb{T}^2 \times s^4$	$\mathbb{T}^2 \times s^5$	black torus
			$S^3 \times s^2$ $\mathbb{T}^3 \times s^2$	$S^3 \times s^3$ $\mathbb{T}^3 \times s^3$	$S^3 \times s^4$ $\mathbb{T}^3 \times s^4$	
				$S^1 \times S^3 \times s^2$	$S^1 \times S^3 \times s^3$ $\mathbb{T}^4 \times s^3$	

for product odd-sphere and even-ball blackfolds with equal sizes and angular momenta (at fixed mass):

$$s(j) \sim j^{-\frac{p}{D-p-3}}$$

$\Rightarrow$ tori dominate entropically

Blackfolds in supergravity and string theory

Emparan,Harmark,Niarchos,NO
Caldarelli,Emparan,v.d.Pol
Grignani,Harmark,Marini,NO,Orselli

- BF method originally developed for neutral BHs, but even richer dynamics when considering charged branes

- simplest case: curving the fundamental black branes of string/M-theory into black holes with with compact horizon topologies (F1, Dp, NS5, M2, M5)

more generally: consider dilatonic black branes that solve action

$$I = \frac{1}{16\pi G}\int d^D x\,\sqrt{g}\left(R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2(p+2)!}e^{a\phi}F^2_{(p+2)}\right)$$

First: p-branes that carry (p+1)-form charge

(p=0: particle charge, p=1: string dipole charge, etc.)

+ important generalizations when having furthermore dissolved charges (anistropic charged fluids), e.g. using multi-charge black branes

notation: spacetime $X^\mu,\ \mu,\nu\ldots = 0,\ldots,D-1$

worldvolume $\sigma^a,\ a,b\ldots = 0,\ldots,p$

$$n = D - p - 3$$

Effective worldvolume theory – leading order

similar to effective theories for other extended objects:

difference: - short-distance d.o.f. = gravitational short-wavelength modes

- extended objects posses black hole horizon

start from flat black brane solution:

- identify symmetries + conserved charges (and 0-modes from SSB): energy, momentum, charges, …
- construct worldvolume densities as functions of these: $T_{\mu\nu},\ J_\mu, \ldots$ (integrate thickness)
- promote collective coordinates to slowly varying quantities

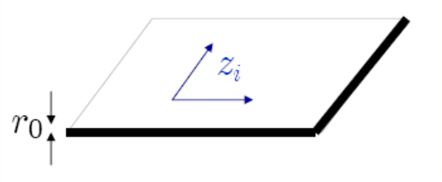
$$X^\mu(\sigma), \varepsilon(\sigma), u^\mu(\sigma), q(\sigma), \ldots$$

positions transverse to worldvolume

energy density (horizon thickness)

local boost velocity

charge

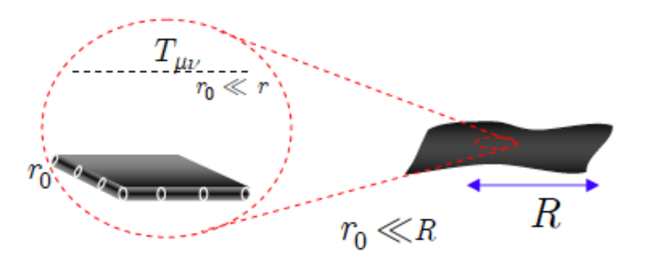


- coordinates $\sigma^a = (t, z^i)$ on brane worldvolume

- equations of motion are:

$$\bar{\nabla}_\mu T^{\mu\nu} = 0, \bar{\nabla}_\mu J^\mu = 0, \ldots$$

validity:



Blackfold dynamics

consistent coupling of wv. to long-wave length gravitational field

$\Rightarrow$ $\overline{\nabla}_\mu T^{\mu\rho} = 0$ (stress tensor supported on worldvolume)

$$\downarrow$$

$$T^{\mu\nu} K_{\mu\nu}{}^{\rho} + \partial_b X^\rho D_a T^{ab}$$

(using embedding tensors, extrinsic curvature tensor etc. etc.)

(for particle worldline: $m a^\nu + (D_\tau m) u^\mu = 0$)

- stress tensor conservation:

$$T^{\mu\nu} K_{\mu\nu}{}^{\rho} = 0 \qquad \text{extrinsic equations (D-p-1)}$$
$$D_a T^{ab} = 0 \qquad \text{intrinsic equations (p+1)}$$

$\Rightarrow$ determine the collective brane coords.

generalized geodesic equation

$$T^{ab}\left(D_a \partial_b X^\rho + \Gamma^\rho_{\mu\nu} \partial_a X^\mu \partial_b X^\nu\right) = 0$$

- charge conservation:

$$D_a J^{a_1 \cdots a_p + 1} = 0$$

Perfect fluid with conserved p-brane charge

perfect fluid with conserved p-brane charge

$$T^{ab} = (\varepsilon + P)u^a u^b + P\gamma^{ab}$$

$$J = Q_p \hat{V}_{(p+1)}$$

$$d\varepsilon = \mathcal{T} ds, \qquad \varepsilon + P = \mathcal{T} s,$$

local potential $$\Phi_p(\sigma^a) = \frac{\partial \varepsilon(s; Q_p)}{\partial Q_p}$$

blackfold equations

- intrinsic: $D_a T^{ab} = 0$ $\quad \partial_a Q_p = 0\,.$
- extrinsic $-PK^\rho = \perp^\rho{}_\mu\, s\mathcal{T}\dot{u}^\mu$

- blackfolds represent objects with horizon
  - reflected in effective theory in entropy and local thermodynamics
- assume regularity of event horizon under long-wave length perturbations when blackfold equations are satisfied
- no rigorous derivation but significant evidence:
  - thin black rings, black tori
    (first correction computed using MAE = matched asymptotic expansion)
  - cf. black branes in AdS

Effective fluid

for charged dilatonic p-branes:
use exact sugra solution to read off the properties of the fluid:

$$\varepsilon = \frac{\Omega_{(n+1)}}{16\pi G} r_0^n \left(n + 1 + nN \sinh^2\alpha\right), \qquad P = -\frac{\Omega_{(n+1)}}{16\pi G} r_0^n \left(1 + nN \sinh^2\alpha\right),$$

$$\mathcal{T} = \frac{n}{4\pi r_0 \left(\cosh\alpha\right)^N}, \qquad s = \frac{\Omega_{(n+1)}}{4G} r_0^{n+1} \left(\cosh\alpha\right)^N,$$

$$Q_p = \frac{\Omega_{(n+1)}}{16\pi G} n\sqrt{N} r_0^n \sinh\alpha \cosh\alpha, \qquad \Phi_p = \sqrt{N} \tanh\alpha .$$

functions of r_0 and α, N = 1,2,3 (depending on case)

stress tensor
takes form:

$$T_{ab} = \mathcal{T} s \left(u_a u_b - \frac{1}{n}\gamma_{ab}\right) - \Phi_p Q_p \gamma_{ab}$$

near extremality:

$$T_{ab}^{(\mathrm{exc})} \simeq \mathcal{T} s \left(u_a u_b + \left(\frac{N}{2} - \frac{1}{n}\right)\gamma_{ab}\right)$$

Stationary solutions

◆equilibrium configurations stationary in time = stationary black holes

$$u = \frac{k}{|k|}\,, \qquad \nabla_{(\mu} k_{\nu)} = 0\,, \qquad \mathcal{T}(\sigma^a) = \frac{T}{|k|} \qquad k = \xi + \Omega\chi$$

can solve intrinsic blackfold equations explicitly (e.g. for thickness and velocity)
-> only need to solve extrinsic equations for the embedding

$$r_0 = \frac{n\sqrt{1-V^2}}{2\kappa} \qquad V^2 = \sum_i \Omega_i^2 R_i^2(\sigma)$$

velocity field

• blackfolds with boundaries: fluid approaches speed of light at bdry. (horizon closes off !)

extrinsic equations: $K^\rho = \perp^{\rho\mu} \partial_\mu \ln(-P)$

⇨ derivable from action $\tilde{I} = \int_{\mathcal{W}_{p+1}} d^{p+1}\sigma \sqrt{-\gamma}\, P$

- thermodynamics: all global quantities: mass, charge, entropy, chemical potentials by integrating suitable densities over the worldvolume

Action principle for stationary blackfolds and 1st law

- for any embedding (not nec. solution) the "mechanical" action is proportional to Gibbs free energy:

$$\beta^{-1} I = G = M - \sum_i \Omega_i J_i - TS$$

- compute mass and angular momentum by integrating appropriate stress tensor components over brane worldvolume + entropy from total area

varying G -> 1st law of thermodynamics

$$dM = TdS + \Omega dJ \qquad (\text{fixed } Q_p)$$

⇨ 1st law of thermo = blackfold equations for stationary configurations

- can also use Smarr relation to show that: total tension vanishes for stationary blackfolds

$$(D-3)M - (D-2)(TS + \Omega J) - n\Phi_H^{(p)} Q_p = \mathcal{T}_{\text{tot}}$$

Odd-sphere blackfolds with dipole p-brane charge

- can solve stationary BF equations on round odd-spheres (most sym. case)

-> find new odd-sphere (+products) stationary black hole solutions
with dipole-like (local) charge
(checks with exact 5D dipole rings (Emparan))

- new stationary black holes in string/M-theory, w. novel horizon topology
- new type of charge (generalizing dipole charge of ring) entering
1^{st} law of thermo (cf. Copsey, Horowitz)
- (presumably) stable for sufficiently high charge (positive specific heat)
- standard extremal limit gives Dirac:

$$T_{ab} = P\gamma_{ab}$$

- interesting new extremal limits with null waves (beyond DBI):

$$T_{ab} = \mathcal{K}\, l_a l_b - \sqrt{N} Q_p\, \gamma_{ab} \qquad l_a l^a = 0$$

obey:

$$\frac{1}{p+1} M = \sqrt{N} V_{(p)} Q_p = \frac{1}{p} \frac{J}{\mathcal{R}}$$

(for 5D fundamental string:
winding=momentum)

Odd-sphere blackfolds in string theory

Emparan,Harmark,Niarchos,NO

Brane (IIA)	Worldvolume	$\perp$ Sphere
F1	S^1	s^7
D2	$\mathbb{T}^2$	s^6
D4	$S^3 \times S^1$, $\mathbb{T}^4$	s^4
NS5	S^5 , $S^3 \times \mathbb{T}^2$	s^3
D6	$S^3 \times S^3$, $S^5 \times S^1$	s^2

Brane (IIB)	Worldvolume	$\perp$ Sphere
D1	S^1	s^7
F1	S^1	s^7
D3	S^3, $\mathbb{T}^3$	s^5
D5	S^5 , $S^3 \times \mathbb{T}^2$	s^3
NS5	S^5 , $S^3 \times \mathbb{T}^2$	s^3

Table 1: *A list of horizon topologies for stationary non-extremal black holes in type IIA/IIB string theory based on the singly-charged blackfolds of the theory with worldvolumes curved into products of odd-spheres. The s^{n+1} denotes the 'small' sphere in horizon directions orthogonal to the worldvolume. The number ℓ of 'large' odd-spheres spanned by the worldvolume is limited by* (3.11).

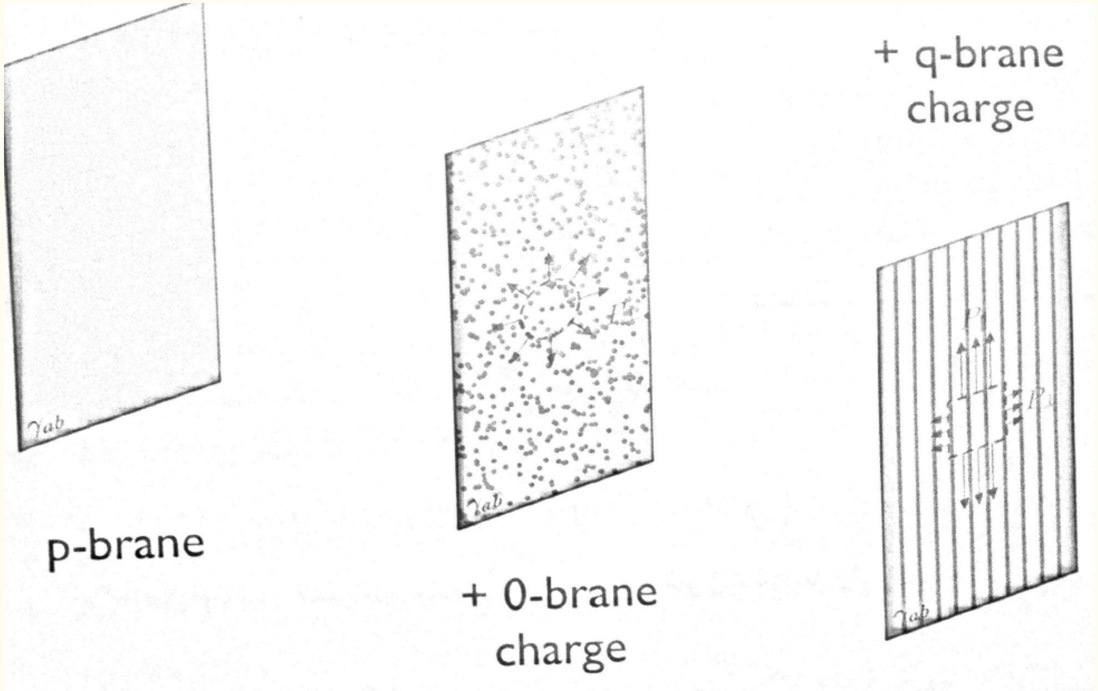
Brane	Worldvolume	$\perp$ Sphere
M2	$\mathbb{T}^2$	s^7
M5	S^5, $S^3 \times \mathbb{T}^2$, $\mathbb{T}^5$	s^4

Table 2: *The analogue of Table 1 in M-theory for M2 and M5 black branes.*

Blackfolds with brane currents

- can perform general analysis of blackfolds with brane currents on them (q=0 and q=1 simplest): anistropic charged perfect fluids (entirely new type of fluid dynamics)
  - is able to capture thermal excitations of e.g. D-branes with lower D-brane or F-string currents

brane currents induce differences in pressures in directions parallel and transverse to them (due to effective tension $\Phi_q Q_q$ along the current)



- charge density conserved along the q-brane but can redistribute itself in transverse directions

- stability of charge waves governed by isothermal permittivity

$$\epsilon_q \equiv \left(\frac{\partial \Phi_q}{\partial Q_q}\right)_{Q_p,T}$$

Examples (2 charges)

blackfolds based on 2-charge brane systems: D0-Dp (p=2,4,6) F1-Dp (p >0)

		Worldvolume	$\perp$ Sphere
	F1-D1	(helical) S^1	s^7
D0-D2	F1-D2	$\mathbb{T}^2$	s^6
	F1-D3	S^3 , $\mathbb{T}^3$	s^5
D0-D4	F1-D4	$S^3 \times S^1$, $\mathbb{T}^4$	s^4
	F1-D5	S^5 , $S^3 \times \mathbb{T}^2$	s^3
D0-D6	F1-D6	$S^3 \times S^3$, $S^5 \times S^1$	s^2

have non-trivial extremal limits:

- extremal branes with subluminal worldvolume velocity
  (boost along direction perpendicular to at least one of currents)

- null-wave branes
  (only when current is parallel to the wave)

Also many interesting cases with charged q=0 or 1 branes dissolved in neutral p-brane
- electric rotating black holes (near-extr, w. slow rotation, new instabilities)
- new classes of black holes with string dipoles

Caldarelli,Emparan,v.d.Pol

Examples (3 charges)

3-charge example: D1-D5-P (e.g. in D=6: S^1 x s^3 horizon)
with finite entropy in extremal limit

could be first example of stable, asympotically flat, extremal, non-supersymmetric brane in ST with non-spherical horizon topology in D > 5

(instability might occur via coupling of intrinsic/extrinsic perturbations)

Dimension (non-compact)	Worldvolume	$\perp$ Sphere
$D = 10$	$S^5\,,\quad S^3 \times \mathbb{T}^2$	s^3
$D = 9$	$S^3 \times S^1\,,\quad \mathbb{T}^4$	s^3
$D = 8$	$S^3\,,\quad \mathbb{T}^3$	s^3
$D = 7$	$\mathbb{T}^2$	s^3
$D = 6$	S^1	s^3

Table 4: *A list of horizon topologies for stationary extremal rotating black holes with D1-D5 dipoles in a spacetime with D non-compact dimensions and $10 - D$ compact KK circles. We do not distinguish whether the D1-P current wraps some of the compact directions, which gives different kinds of black holes.*

other new extremal black holes in ST ?

Gregory-Laflamme instability

- blackfold approach captures perturbative dynamics of BH when $\lambda \gg r_0$

  - can be intrinsic variations (thickness, local velocity) or extrinsic (embedding)
    -> generally coupled

simple case: $r_0 \ll \lambda \ll R$ -> worldvolume looks flat

- decoupling between intrinsic/extrinsic

for a general perfect fluid:

- transverse (elastic) perturbations $$c_T^2 = -\frac{P}{\varepsilon}$$

- longitudinal (soundmode) perturbations $$c_L^2 = \frac{dP}{d\varepsilon}$$

p-brane blackfolds have:

$$c_s^2 = -\frac{1}{n+1}\frac{1+(2-Nn)\sinh^2\alpha}{1+\left(2-\frac{Nn}{n+1}\right)\sinh^2\alpha}$$

(can become positive)

- sound mode instability is long-wavelength part of GL instability !

$$\delta r_0 \sim e^{\Omega t + i k_i z^i}$$

$$\Omega = \frac{1}{\sqrt{n+1}}k\,.$$

good agreement with slope of GL curve

also: $$\Omega = \sqrt{\frac{s}{|c_v|}}\,k$$

cf. correlated stability conjecture (Gubser, Mitra)

Further connections (w. some recent work, not exhaustive!)

-new charge and/or dipole rings:
Rocha,Rodriguez.Virmani/Compere, de Buyl, Stotyn,Virmani/ Rodriguez
Kleihaus, Kunz, Schnulle
Bena, Giusto, Ruef

- near-horizons analysis of supersymmetric non-spherical BHs
  Kunduri, Lucietti/Gutowski, Papadapoulos, ….

- new instabilities of spinning BHs
  Dias,Figueras,Monteiro,Santos,…

- domain structure (generalizing rod-structure) and horizon topologies
  Harmark/Armas,Harmark,Caputa

- self-similar critical geometries at horizon intersections and mergers
  Kol/Emparan,Haddad

- response coefficients for BHs (Love numbers)   Kol

- 3-charge microstate geometries
  Bena,de Boer,Shigemori,Warner

Outlook

• charged blackfolds with multiple charges/extremal limits
  - developed general theory of anistropic p'-form charged fluids
  - opens up new interesting dynamics (cf. supertubes)
  - new extremal black holes in ST (microscopics ?)

• method can also be applied to blackfolds in other backgrounds (AdS, dS)
  + turning on other fields
  - black rings in (A)dS
  - blackfolds in (A)dS Caldarelli,Empran,Rodriguez
  Armas, NO

• duality of higher D black holes to plasma balls + rings in SS AdS
  (cf. Lahiri,Minwalla) – many similar features

• relation to fluid/gravity correspondence (recently: Wilsonian approach)

• higher-order analysis (beyond pole approximation, dipole effects,
  intrinsic spin, MAE/CIEFT)
  Armas, Camps, Harmark, NO

• perturbations/stability analysis (new response coefficients,
  combined intrinsic/extrinsic perturbations, time-dependent BF)

• further elucidate relation of BFs with DBI/NG

Outlook (con’td)

- blackfold gives a new method for D-brane (and other) probes in thermal backgrounds

+ applied to simplest case: Bion in hot flat space

- talk of TH: finite T brane/anti-brane wormhole configuration of BIon: behaves qualitatively different from zero-temperature counterparts

■ contrary to previous work:
- takes into account that the probe itself is a thermal object

□ apply new perspective to AdS probes (thermal AdS or AdS BH)
- may resolve discrepancies between gravity and gauge theory found for Polaykov loops based on D3/D5
- revisit other previously studied cases

-> Polyakov loops using thermal fundamental string probes in AdS/CFT.

Grignani, Harmark, Marini,NO ,Orselli (to appear)