

Black hole microstate geometries from string amplitudes

David Turton

Queen Mary, London

PICS meeting, 18 Oct 2010

Based on

arXiv:1007.2856 W. Black, R. Russo, DT
& work in progress

Message

- The non-trivial features of the supergravity fields corresponding to various D-brane bound states can be calculated from string amplitudes.
- This allows us to identify certain supergravity solutions as gravitational descriptions of particular microscopic bound states of strings and D-branes.
- This identification can shed light on possible descriptions of BPS black holes in supergravity.

Outline

- 1 Review of some open problems in black hole physics and attempts to address them in string theory and supergravity
- 2 Two-charge geometries from string amplitudes
 - D1-D5 duality frame
 - D1-P duality frame
- 3 Three-charge geometries from string amplitudes (work in progress)
- 4 Outlook and open questions

Motivation: open problems in black hole physics

Some important outstanding problems black hole physics
(related to each other):

- 1 Black holes have entropy proportional to their area, conflicting with black hole uniqueness theorems. Can this entropy be given a statistical interpretation in general?
- 2 Is black hole formation and evaporation a unitary process?
- 3 Is information preserved or destroyed in black hole evaporation?

Some terminology

- In this talk, by a 'black hole' I will mean a solution to general relativity or supergravity (or higher-derivative generalizations) with an event horizon surrounding a totally collapsed body, e.g. Schwarzschild, Kerr, Reissner-Nordstrom. (Avoids using terms such as 'mathematical black hole' and '(astro)physical black hole')
- A black hole solution is, in some approximation \mathcal{A} , the gravitational field sourced by a particular **heavy, compact bound state of matter**.
- A prevailing perspective (at least among string theorists) is that a black hole solution should be a thermodynamic description of an ensemble of microscopic degrees of freedom.

Microscopics of black holes

In string theory, the microscopic degrees of freedom can be described in many ways (depending on context):

- As excitations of a wrapped fundamental string

Sen

- As degrees of freedom of a D-brane worldvolume theory

Strominger, Vafa

- Using AdS/CFT

Lunin, Mathur
Skenderis, Taylor

- A (possibly large) subset of degrees of freedom may be well described by families of smooth regular supergravity solutions.

Mathur
Bena, Warner

Microscopics of black holes

A conjecture about black hole physics of particular interest is:

- The **'fuzzball'** conjecture: the conjecture that the microscopic degrees of freedom alter the physics described by the black hole solution in the region of spacetime around the event horizon of the black hole solution, in particular the physics of Hawking radiation.

Mathur & collaborators

An important role in providing evidence for this conjecture has come from families of smooth regular supergravity solutions, differing at the horizon scale, which have been called **'microstate geometries'**.

Bena, Warner & collaborators

An important question is how large a subset of degrees of freedom may be well described by such smooth regular supergravity solutions.

Role of smooth solutions to supergravity

- Given a regular, smooth, horizonless solution to supergravity with the same charges as a black hole, can this be associated to a microscopic degree of freedom of a black hole or is it something else?
- One possible interpretation is that all such solutions should be regarded as ‘hair’ and counted separately to the black hole solution when determining the entropy of a given system
- Should **any** smooth solutions have interpretations as ‘microstates’? If so, which ones?

Sen

Role of smooth solutions to supergravity

- If any given solution is to be associated to a degree of freedom of a black hole rather than 'hair', there should be a logical connection, e.g.
 - 'Scaling solutions' have similar throats to the black hole solution
Bena, Wang, Warner
 - Our calculations can associate a supergravity solution with a microscopic D-brane bound state, with given resolution.
- Not yet clear whether smooth solutions alone can, even in principle, be numerous enough to account for the entropy of a black hole; non-geometric solutions (U-folds, etc) may have a role to play.

de Boer & collaborators

Context of our work

- Our amplitudes reproduce the **asymptotic** form of the supergravity fields sourced by a D-brane bound state, perturbatively in g_s and $1/r$. Note that this is the full asymptotically flat solution, not the near-horizon AdS region.
- From the amplitudes themselves, one cannot determine whether a solution is everywhere smooth and regular; the full solution can in principle be reconstructed from our linearized solution by using the full non-linear supergravity equations of motion.
- In our two-charge calculations, there is no black hole solution in the duality frames we work in; the three-charge D1-D5-P system however has a black hole solution with macroscopic horizon.
- In all cases, for large charges we will be calculating the gravitational fields sourced by **heavy, compact bound states of strings and branes**.

2 Two-charge geometries from string amplitudes

D-brane fields in IIB supergravity

We shall work in type IIB theory on $\mathbb{R}_t \times S^1 \times \mathbb{R}^4 \times T^4$.

Indices: $i = 1, \dots, 4$ (\mathbb{R}^4), $a = 5, \dots, 8$ (T^4), $A = 1, \dots, 8$.

Consider for example a static D1-brane wrapped on S^1 , and smeared on T^4 . The corresponding supergravity solution is

$$\begin{aligned} ds^2 &= H^{-\frac{1}{2}}(-dt^2 + dy^2) + H^{\frac{1}{2}} dx^A dx^A; \\ e^{2\hat{\Phi}} &= H; \quad C_{ty}^{(2)} = -(H^{-1} - 1) \end{aligned}$$

where

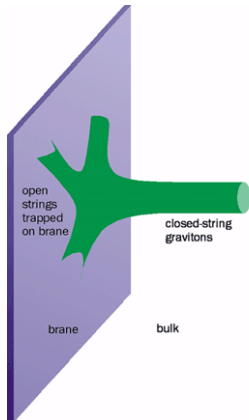
$$H = 1 + \frac{Q_1}{|x_i|^2}.$$

D-brane supergravity fields from IIB string theory

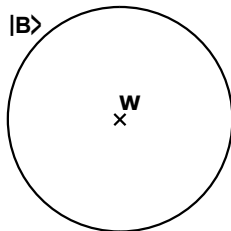
D-brane solutions like the one on the previous slide may be reproduced by calculating a disk one-point function for closed string emission.

Di Vecchia, Frau, Pesando, Sciuto, Lerda, Russo '97

- Spacetime cartoon:



- Worksheet diagram:



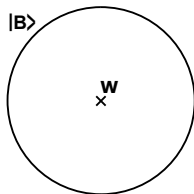
D-brane supergravity fields from IIB string theory

The calculation of the supergravity fields from disk one-point functions has the following recipe (take graviton for example):

- 1 Calculate the amplitude for emission of NS-NS massless closed string states with momentum k^i in the non-compact directions only:

$$\mathcal{A}_{\text{NS}}(k^i)$$

- 2 Extract the field of interest: $h_{\mu\nu}(k^i) = \frac{\delta \mathcal{A}_{\text{NS}}}{\delta h^{\mu\nu}}$
- 3 Multiply by the free propagator $\frac{1}{|k_i|^2}$
- 4 Fourier transform in the non-compact directions to obtain the spacetime field $h_{\mu\nu}(x_i)$.



D1-brane with momentum - supergravity

One can add a momentum charge in the y direction by adding an extra term in the metric: $(v = t - y, \quad u = t + y)$

$$ds^2 = H^{-\frac{1}{2}}(-dudv + Kdv^2) + H^{\frac{1}{2}}dx^A dx^A$$

with the dilaton and RR field as before, and

$$K = \frac{Q_p}{|x_j|^2} .$$

While this is a BPS solution to supergravity, there is no known microscopic configuration in string theory which can source this geometry.

Physically this is due to the fact that the momentum must be carried by **transverse** oscillations of the D-brane, in the form of a null travelling wave.

D1-brane with momentum - supergravity

There is a family of solutions representing the fields sourced by a D-brane with a travelling wave; they are S-dual to the supergravity solutions for a fundamental string with winding and momentum.

Dabholkar, Gauntlett, Harvey, Waldram '95

The fields are

$$\begin{aligned} ds^2 &= H^{-\frac{1}{2}}(-dudv + Kdv^2 + A_I d v dx^I) + H^{\frac{1}{2}} dx^A dx^A \\ e^{2\hat{\phi}} &= H; \quad C_{uv}^{(2)} = -(H^{-1} - 1); \quad C_{vI}^{(2)} = -H^{-1} A_I; \end{aligned}$$

where the harmonic functions take the form
(\hat{v} is the worldvolume light-cone coordinate)

$$H = 1 + \frac{Q_1}{L} \int_0^L \frac{d\hat{v}}{|x_i - f_i(\hat{v})|^2}, \quad A_I = -\frac{Q_1}{L} \int_0^L \frac{d\hat{v} \dot{f}_I(\hat{v})}{|x_i - f_i(\hat{v})|^2}, \quad K = \frac{Q_1}{L} \int_0^L \frac{d\hat{v} |\dot{f}_I(\hat{v})|^2}{|x_i - f_i(\hat{v})|^2}.$$

The solutions are singular at the brane location $x_i = f_i(\hat{v})$.

D1-D5 fields in IIB supergravity

We dualize to the D1-D5 duality frame, where the solutions smooth and regular. Indices are split $f_A(v) \equiv (f_i(v), f_{\hat{a}}(v), f(v))$ where \hat{a} runs over the three self-dual two-forms on the T^4 and the remaining component is the preferred direction in the T^4 picked out by the dualities.

The metric and B-field are then given by

$$ds^2 = \frac{\hat{H}_1^{1/2}}{\tilde{H}_1 H_5^{1/2}} [-(dt - A_i dx_i)^2 + (dy + B_i dx_i)^2] + (\hat{H}_1 H_5)^{1/2} dx_i dx_i + \left(\frac{\hat{H}_1}{H_5}\right)^{1/2} dx_a dx_a,$$

$$b = -\frac{A}{\tilde{H}_1 H_5} (dt - A) \wedge (dy + B) + B + \frac{A_{\hat{a}} \omega^{\hat{a}}}{H_5},$$

where

$$H_5 = 1 + \frac{Q_5}{L} \int_0^L \frac{dv}{|x_i - f_i(v)|^2}, \quad H_1 = 1 + \frac{Q_5}{L} \int_0^L \frac{dv |\dot{f}_A(v)|^2}{|x_i - f_i(v)|^2},$$

$$\hat{H}_1 = H_1 - \frac{A_{\hat{a}} A_{\hat{a}}}{f_5}, \quad \tilde{H}_1 = H_1 - \frac{A_{\hat{a}} A_{\hat{a}} + \mathcal{A} \mathcal{A}}{f_5},$$

$$A_A = -\frac{Q_5}{L} \int_0^L \frac{dv \dot{f}_A(v)}{|x_i - f_i(v)|^2} \equiv (A_i, A_{\hat{a}}, \mathcal{A}), \quad A \equiv A_i dx_i,$$

$$dB = -*_4 dA, \quad dB = *_4 d\mathcal{A}.$$

D1-D5 fields in supergravity - asymptotic expansion

The D1-D5 solutions differ from the naive D1-D5 geometry at order $1/r^3$.

The large distance expansion of A_i and \mathcal{A} is given by

$$A_i \approx -\frac{Q_5}{L} \int_0^L dv \dot{f}_i \left[\frac{1}{r^2} + 2 \frac{x_j f_j}{r^4} \right] = -2Q_5 \hat{f}_{ij} \frac{x_j}{r^4}, \quad \hat{f}_{ij} = \frac{1}{L} \int_0^L dv \dot{f}_i f_j = -\hat{f}_{ji},$$

$$\mathcal{A} \approx -\frac{Q_5}{L} \int_0^L dv \dot{f} \left[\frac{1}{r^2} + 2 \frac{x_j f_j}{r^4} \right] = -2Q_5 \hat{f}_j \frac{x_j}{r^4}, \quad \hat{f}_j = \frac{1}{L} \int_0^L dv \dot{f} f_j,$$

and the remaining $1/r^3$ terms of the NS-NS fields are

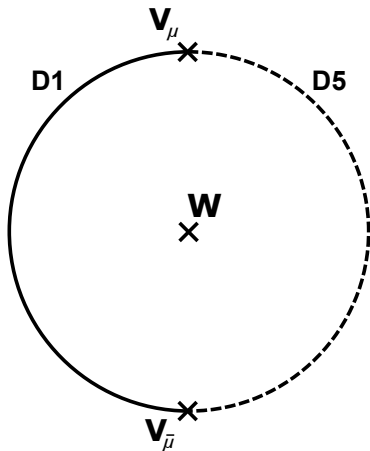
$$g_{ti} = -\frac{2Q_5 x_j \hat{f}_{ij}}{r^4}, \quad g_{yi} = -\epsilon_{ijkl} \frac{Q_5 x_j \hat{f}_{kl}}{r^4},$$
$$b_{ty} = \frac{2Q_5 x_i \hat{f}_i}{r^4}, \quad b_{ij} = 2\epsilon_{ijkl} \frac{Q_5 x_k \hat{f}_l}{r^4},$$

The same can be done for the R-R fields.

Disk amplitudes for D1-D5 fields

The $1/r^3$ terms above can be reproduced by disk amplitudes with mixed boundary conditions:

Giusto, Morales, Russo '09



Disk amplitudes for D1-D5 fields

The vertex operators appearing in the NS-NS amplitude are:

- Twisted fermion zero mode vertices:

① $V_\mu = \mu^A e^{-\frac{\varphi}{2}} S_A \Delta$

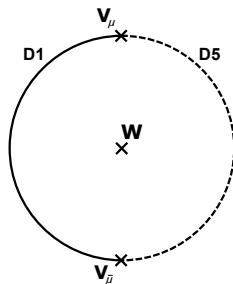
② $V_{\bar{\mu}} = \bar{\mu}^B e^{-\frac{\varphi}{2}} S_B \Delta$

where

- μ^A and $\bar{\mu}^B$ are Chan-Paton matrices encoding the vevs given to the 1-5 and 5-1 open strings
- φ is the free boson of the superghosts (β, γ)
- S_A are the $SO(1,5)$ spin fields
- Δ is the bosonic twist operator which changes the ND boundary conditions.

- NS-NS closed string vertex:

③ $W_{NS}(z, \bar{z}) = \mathcal{G}_{\hat{M}\hat{N}} \left(\partial X_L^{\hat{M}} - ik_L \cdot \psi \psi^{\hat{M}} \right) e^{ik_L X_L(z)} \tilde{\psi}^{\hat{N}} e^{-\tilde{\varphi}} e^{ik_R X_R(\bar{z})}$



Disk amplitudes for D1-D5 fields

The Chan-Paton factors encoding the condensate of R open strings can be expanded as

$$\bar{\mu}^A \mu^B = v_I (C\Gamma^I)^{[AB]} + \frac{1}{3!} v_{IJK} (C\Gamma^{IJK})^{(AB)},$$

and the second term sources the fields we are interested in. It is self-dual:

$$v_{IJK} = \frac{1}{3!} \epsilon_{IJKLMN} v^{LMN}.$$

Disk amplitudes for D1-D5 fields

The amplitude with self-dual condensate sources the fields

$$g_{ti}(x) = -\frac{\sqrt{2}}{\pi} \frac{x^j v_{tij}}{r^4} \quad , \quad b_{ty}(x) = \frac{2\sqrt{2}}{\pi} \frac{x^i v_{tyi}}{r^4} \quad .$$

$$g_{yi}(x) = -\frac{1}{\sqrt{2}\pi} \epsilon_{ijkl} \frac{x^j v_{tkl}}{r^4} \quad , \quad b_{ij}(x) = \frac{2\sqrt{2}}{\pi} \epsilon_{ijkl} \frac{x^k v_{tyl}}{r^4} \quad .$$

Comparing g_{ti} and b_{ty} to supergravity gives the dictionary

$$Q_5 \hat{f}_{ij} = \frac{1}{\sqrt{2}\pi} v_{tij} \quad , \quad Q_5 \hat{f}_i = \frac{\sqrt{2}}{\pi} v_{tyi} \quad .$$

which then gives agreement for g_{yi} and b_{ij} .

Thus this amplitude reproduces the $1/r^3$ terms of the D1-D5 geometries.

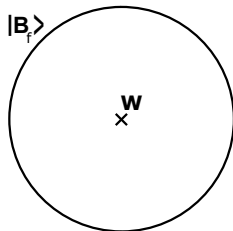
D-brane/momentum duality frame

- In order to extend this technology to the three-charge D1-D5-P system, it is necessary to understand how to include a momentum charge into the setup.
- For this reason, we next study the two charge system in the D1-P (equivalently, D5-P) duality frame.

Black, Russo, DT

D-brane/momentum duality frame

- In this duality frame, there is a technical advantage that one can use boundary state technology to encode the profile of the D-brane exactly (this has the effect of resumming all momentum insertions)
- Using this technology we were able to reproduce the full functional form of the harmonic functions, not just the expansion in $1/r$.



D1-P supergravity solutions

Let us recall the D1-P supergravity solutions:

$$\begin{aligned} ds^2 &= H^{-\frac{1}{2}}(-dudv + A_I dv dx^I + K dv^2) + H^{\frac{1}{2}} dx^A dx^A \\ e^{2\hat{\Phi}} &= H; \quad C_{uv}^{(2)} = -(H^{-1} - 1); \quad C_{vl}^{(2)} = -H^{-1} A_l; \end{aligned}$$

where the harmonic functions are

$$H = 1 + \frac{Q_1}{L} \int_0^L \frac{d\hat{v}}{|x_i - f_i(\hat{v})|^2}, \quad A_I = -\frac{Q_1}{L} \int_0^L \frac{d\hat{v} \dot{f}_I(\hat{v})}{|x_i - f_i(\hat{v})|^2}, \quad K = \frac{Q_1}{L} \int_0^L \frac{d\hat{v} |\dot{f}_I(\hat{v})|^2}{|x_i - f_i(\hat{v})|^2}.$$

We will calculate the amplitudes for h_{vl} and h_{vv} (where $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$); the remaining fields are analogous.

D1-P supergravity solutions - expansion

Expanding h_{VI} and h_{VV} to linear order in $Q_1 \sim g_s$ gives

$$h_{VI} = \frac{Q_1}{L} \int_0^L \frac{-\dot{f}_i d\hat{v}}{|x_i - f_i(\hat{v})|^2}, \quad h_{VV} = \frac{Q_1}{L} \int_0^L \frac{|\dot{f}|^2 d\hat{v}}{|x_i - f_i(\hat{v})|^2}$$

and the disk amplitudes in this duality frame can reproduce this form of the fields, not just the expansion in $1/r$.

Boundary state technology

The boundary state $|B\rangle$ is a BRST invariant state of the closed string that inserts a boundary on the world-sheet and enforces the boundary conditions appropriate for a D-brane.

For a flat Dp -brane these boundary conditions are ($\eta = \pm 1$):

$$\begin{aligned} \partial_\tau X^\parallel|_{\tau=0} &= 0, & (X^\perp - y^\perp)|_{\tau=0} &= 0, \\ (\psi^\parallel - i\eta\tilde{\psi}^\parallel)|_{\tau=0} &= 0, & (\psi^\perp + i\eta\tilde{\psi}^\perp)|_{\tau=0} &= 0. \end{aligned}$$

Expanding in oscillators, the boundary conditions on non-zero modes can be encoded in a reflection matrix $R = \text{diag}(1, \dots, 1, -1, \dots, -1)$ (which is $+$ for \parallel , $-$ for \perp) as follows:

$$\tilde{\psi}_r^\mu = i\eta R^\mu{}_\nu \psi_{-r}^\nu, \quad \tilde{\alpha}_n^\mu = -R^\mu{}_\nu \alpha_{-n}^\nu,$$

and the zero modes must satisfy

$$p^\parallel = 0, \quad x^\perp = y^\perp.$$

Boundary state technology

When we include a momentum charge, we give the D-brane a profile which is an arbitrary travelling wave, $f(\hat{v})$.

This introduces a boundary action, most conveniently written in the D9 frame where the worldvolume gauge field takes the place of the profile function:

$$S_f = i \int_{\partial M} dz \left(A_\mu(X) (\partial X^\mu + \bar{\partial} X^\mu) - \frac{1}{2} (\psi^\mu + \tilde{\psi}^\mu) F_{\mu\nu} (\psi^\nu - \tilde{\psi}^\nu) \right)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$; under T-duality $A_\mu \leftrightarrow f_\mu$.

The boundary conditions on the worldsheet fields are in general non-linear.

Haggi-Mani, Lindstrom, Zabzine '00

The boundary state for a D-brane with a travelling wave has been previously constructed;

Hikida, Takayanagi, Takayanagi; Blum; Bachas, Gaberdiel '03

- we make a simple generalization to the case of higher D-brane wrapping number and calculate the amplitude for emission of classical massless fields.

Boundary state technology

Since the profile is purely left-moving, only the linear terms in the boundary conditions contribute.

We write the oscillator boundary conditions as

$$\tilde{\psi}_r^\mu = i\eta (R_f)^\mu{}_\nu \psi_{-r}^\nu + \dots, \quad \tilde{\alpha}_n^\mu = - (R_f)^\mu{}_\nu \alpha_{-n}^\nu + \dots,$$

where R_f has the lowered-index form

$$(R_f)_{\mu\nu}(v) = \begin{pmatrix} -2|\dot{f}(v)|^2 & -\frac{1}{2} & 2\dot{f}^I(v) \\ -\frac{1}{2} & 0 & 0 \\ 2\dot{f}^I(v) & 0 & -\mathbb{1} \end{pmatrix},$$

where we note $(R_f)_{vI} \sim \dot{f}^I$ and $(R_f)_{v\nu} \sim |\dot{f}(v)|^2$.

The boundary conditions on the zero modes become

$$p_v + \dot{f}^I(v) p_I = 0, \quad p_u = 0, \quad p_a = 0, \quad x^i = f^i(v).$$

Boundary state technology

For our amplitude it suffices to construct the boundary state explicitly for the zero modes and to impose the oscillator boundary conditions by hand.

The boundary conditions on the previous slide are solved by a boundary state whose zero-mode structure in the t , y and x^i direction is is

$$\int dv du \int \frac{d^4 p_i}{(2\pi)^4} e^{-ip_i f^i(v)} |p_i\rangle |u\rangle |v\rangle$$

and we can denote the full boundary state schematically as

$$\int dv du \int \frac{d^4 p_i}{(2\pi)^4} e^{-ip_i f^i(v)} |p_i\rangle |u\rangle |v\rangle |D5; f(s)\rangle_{X,\psi} .$$

Boundary state technology

A D1-brane wrapped n_w times may be viewed as a collection of n_w different D-brane strands with a non-trivial holonomy gluing these strands together.

Restricting to the sector of closed strings with trivial winding (m) and Kaluza-Klein momentum (k), the full boundary state is simply the sum of the boundary states for each constituent, with the condition that the value of f at the end of one strand must equal the value of f at the beginning of the following strand, given by:

$$|D1; f\rangle^{k,m=0} = \sum_{s=1}^{n_w} \int du \int_0^{2\pi R} dv \int \frac{d^4 p_i}{(2\pi)^4} e^{-ip_i f_{(s)}^i(v)} |p_i\rangle |u\rangle |v\rangle |D1; f_{(s)}\rangle_{X,\psi}^{k,m=0}.$$

Disk amplitudes for D1-P fields

The sum over strands transforms the integral over ν to an integral over the full worldvolume coordinate $\hat{\nu}$.

The amplitude for NSNS closed string emission is then (suppressing prefactors)

$$\mathcal{A}_{\text{NS}}^{(\eta)}(k) \equiv \langle 0; k^i | \mathcal{G}_{\mu\nu} \psi_{\frac{1}{2}}^{\mu} \tilde{\psi}_{\frac{1}{2}}^{\nu} | D1; f \rangle = \frac{1}{L_T} \int_0^{L_T} d\hat{\nu} e^{-ik_i f^i(\hat{\nu})} \mathcal{G}_{\mu\nu} R_f^{\nu\mu}(\hat{\nu}) .$$

Disk amplitudes for D1-P fields

Inserting the free propagator and Fourier transforming then gives the denominator of the harmonic functions (which was inaccessible in the D1-D5 calculation):

$$\int \frac{d^4 k}{(2\pi)^4} \frac{e^{ik^i(x^i - f^i)}}{k^2} = \frac{1}{4\pi^2} \frac{1}{|x^i - f^i|^2}$$

and the components of the modified reflection matrix bring the factors of \dot{f} , giving the result

$$h_{vl} = \frac{Q_1}{L} \int_0^L \frac{-\dot{f}_l d\hat{v}}{|x_i - f_i(\hat{v})|^2}, \quad h_{vv} = \frac{Q_1}{L} \int_0^L \frac{|\dot{f}|^2 d\hat{v}}{|x_i - f_i(\hat{v})|^2}$$

and the other fields follow similarly.

- ③ Three charge geometries from string amplitudes
(work in progress)

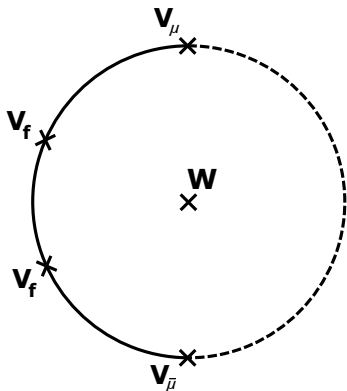
Three-charge geometries in supergravity

There is by now a large literature on three-charge supergravity solutions. Among them are:

- General classification of three-charge geometries with Gibbons-Hawking bases
Bena, Warner '04,...
- Solutions derived from taking extremal limits of non-extremal three charge solutions
Mathur, Giusto, Saxena '04
- Solutions derived from spectral flow from the D1-D5 system
Lunin '04
- Solutions derived from limits of four-charge solutions
Bena, Kraus '05
- Non-extremal D1-D5-P geometries have also been constructed.
Jejjala, Madden, Ross, Titchener '05

Three-charge amplitudes

We are currently studying three-charge amplitudes. The simplest thing to do is to encode the momentum perturbatively:



We have some very preliminary but encouraging early results - work is in progress.

Summary

- Two-charge supergravity solutions can be derived from disk amplitudes
 - In the D1-D5 duality frame, the leading order deviations in g_s and $1/r$ from the naive geometry have been reproduced.
 - In the D1-P duality frame, we reproduced the leading order terms in g_s , to all orders in $1/r$.
- This provides a direct link between these geometries and the microscopic bound states which source them.
- Work is in progress on extending these calculations to the three-charge system.

Open questions

- How much data can we extract from the three-charge amplitudes?
- Can we include the momentum exactly as we did in the D1-P system?
- How many known smooth three-charge geometries correspond to true bound states?
- Can this data be used as input for constructing larger families of smooth three-charge geometries?

Thanks!