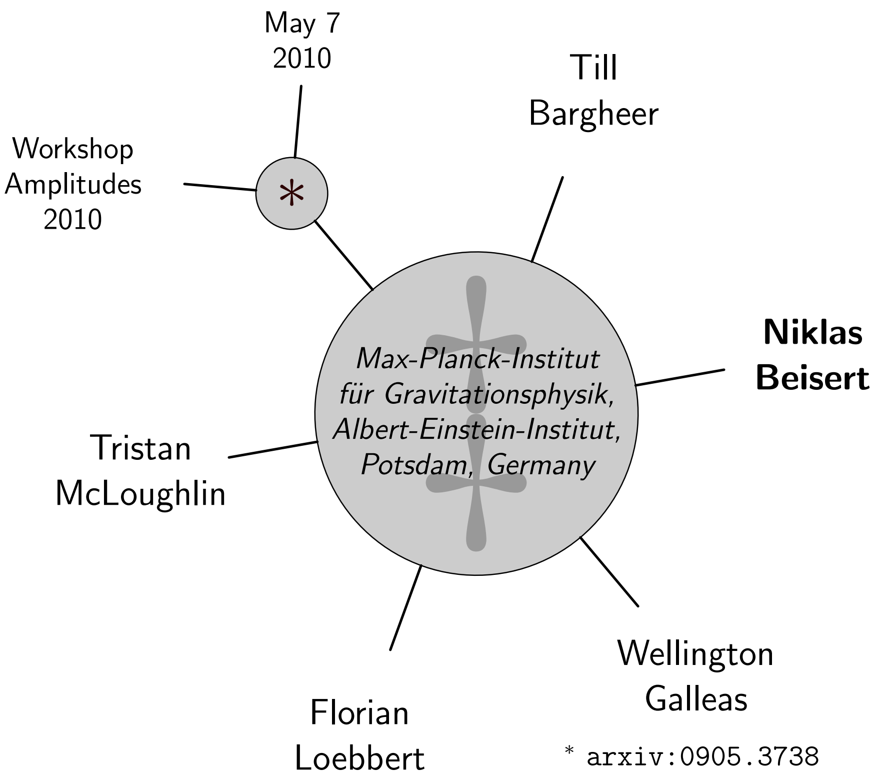
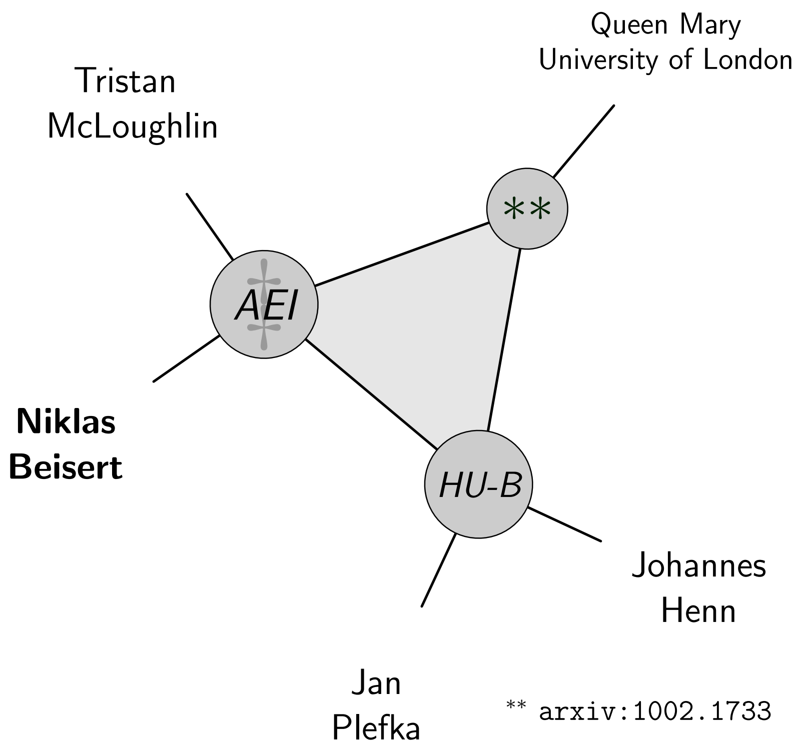
Conformal Symmetry and Integrability in Perturbative $\mathcal{N} = 4$ Scattering Amplitudes*,**





* `arxiv:0905.3738`

** `arxiv:1002.1733`

Scattering Amplitudes in $\mathcal{N} = 4$ SYM

Remarkable $\mathcal{N} = 4$ Super Yang–Mills:

- Exact superconformal symmetry $\mathfrak{psu}(2,2|4)$.
- And some mysterious features: AdS/CFT, integrability, T-duality, ...
- Integrability in the planar limit: $\mathfrak{psu}(2,2|4)$ Yangian.
- Simplifications for planar scattering.
- Duality of (some) amplitudes and Wilson loops.

Questions

- Same cusp dimension from amplitudes & integrable system: How to apply integrability to scattering amplitudes?
- Can one compute remainder function $F(p,\lambda)$ (like $D_{\text{cusp}}(\lambda)$)?
- Relation between (dual) superconformal symmetry and integrability?
- What about non-MHV amplitudes?
- What about non-planar corrections?

Outline

Symmetries of Scattering Amplitudes (S-matrix):

- Understand symmetries of S-matrix.
- Apply symmetries to (fully?) constrain S-matrix.

How to treat (super)conformal symmetry of the S-matrix in $\mathcal{N} = 4$ SYM?

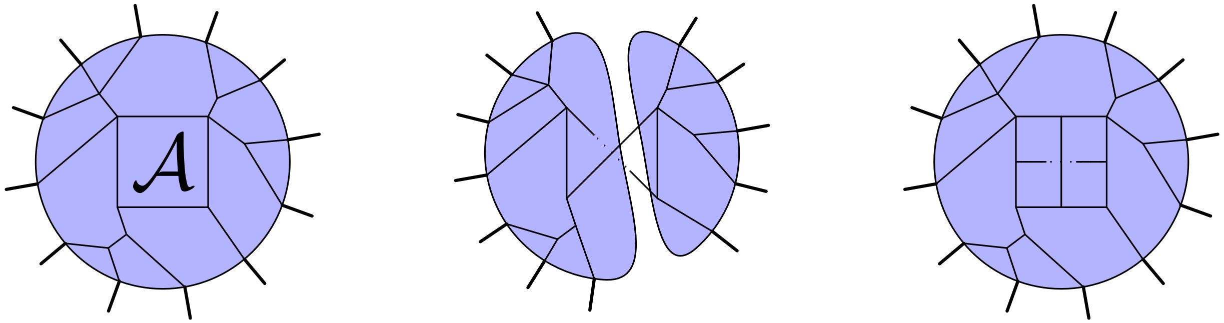
Concretely

- Structure of symmetries: superconformal and Yangian
- Free symmetries
- Symmetries at tree level
- Symmetries at one loop

Free Symmetries

Scattering Amplitudes

Colour-ordered scattering amplitudes (1-trace, 2-trace, genus-1):



Legs: Field Ω combines on-shell gluons Γ, fermions Ψ & scalars Φ:

$$\Omega(\lambda, \tilde{\lambda}, \bar{\eta}) = \Gamma(\lambda, \tilde{\lambda}) + \bar{\eta}^a \Psi_a(\lambda, \tilde{\lambda}) + \tfrac{1}{2}\bar{\eta}^a \bar{\eta}^b \Phi_{ab}(\lambda, \tilde{\lambda}) + \ldots .$$

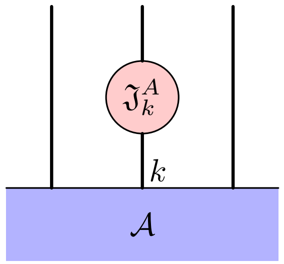
Amplitude $\mathcal{A}(\Lambda_1, \ldots, \Lambda_n)$ on spinor helicity superspace $\Lambda = (\lambda, \tilde{\lambda}, \bar{\eta})$

$$p^{\beta\dot{\alpha}} = \lambda^\beta \tilde{\lambda}^{\dot{\alpha}}, \qquad q^{\alpha b} = \lambda^\beta \bar{\eta}^a.$$

Free Superconformal Symmetry

Free representation $\mathfrak{J}$ of superconformal algebra $\mathfrak{psu}(2,2|4)$:

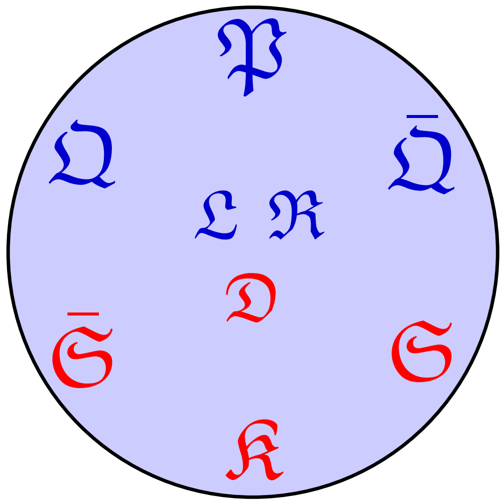
$$\mathfrak{J} = \sum_{k=1}^{n}$$



- Single-leg representation $\mathfrak{J}_k$: multiplet of free on-shell fields Ω.
- Symmetry: Amplitudes are invariant under $\mathfrak{J}$.
- Representation independent of colour structure.

Algebra Generators: (Einfuehrung in Fraktur)

- super-Poincaré: $\mathfrak{P}$, $\mathfrak{L}$, $\mathfrak{Q}$, $\bar{\mathfrak{Q}}$, $\mathfrak{R}$
- superconformal: $\mathfrak{K}$, $\mathfrak{D}$, $\mathfrak{S}$, $\bar{\mathfrak{S}}$

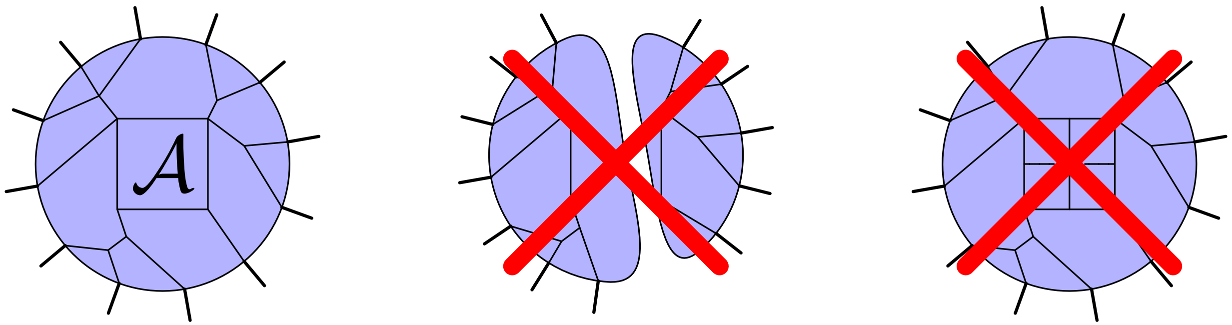


Invariance of tree (N*MHV) amplitudes:

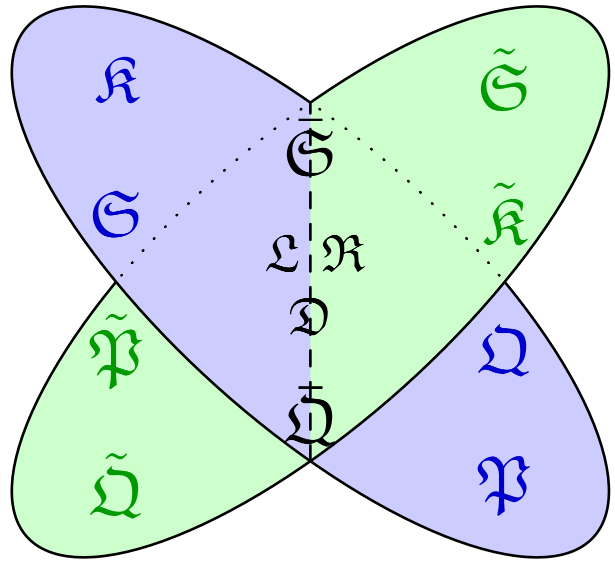
- $\mathfrak{P}$, $\mathfrak{Q}$: manifest through delta functions,
- $\mathfrak{L}$, $\mathfrak{R}$, $\mathfrak{D}$: weight and index contractions,
- $\bar{\mathfrak{Q}}$, $\bar{\mathfrak{S}}$: medium difficulty,
- $\mathfrak{K}$, $\mathfrak{S}$: hardest.

Dual Superconformal Symmetry

Remarkable features of disk amplitudes (single-trace, large-N_c):



- Simplifications: BDS formula.
- Only particular integrals appear.
  Dual conformal symmetry? [Drummond Korchemsky Sokatchev]
- Dual superconformal symmetry! [Drummond, Henn Korchemsky Sokatchev]
- Another $\mathfrak{psu}(2,2|4)$:
  $\mathfrak{L}$, $\mathfrak{R}$, $\mathfrak{D}$, $\bar{\mathfrak{Q}}$, $\bar{\mathfrak{S}}$: shared with conventional,
  $\tilde{\mathfrak{P}}$, $\tilde{\mathfrak{Q}}$: trivial new generators,
  $\tilde{\mathfrak{K}}$, $\tilde{\mathfrak{S}}$: non-trivial new generators.



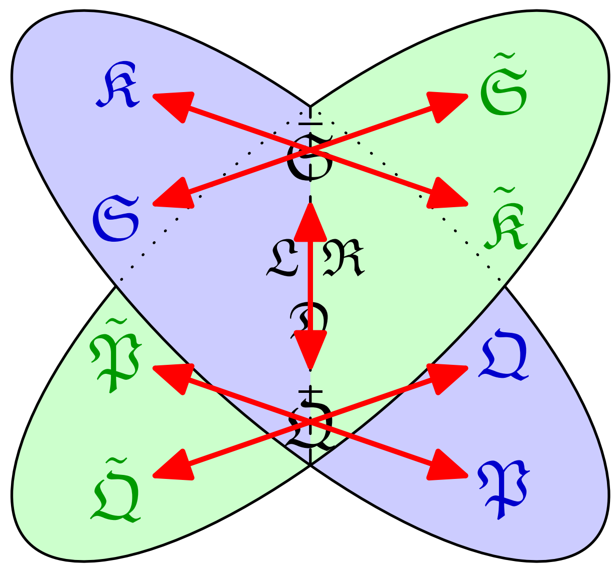
T-Self-Duality in String Theory

Detour: Consider string theory picture at strong coupling.

- Strings propagate on $AdS_5 \times S^5$ superspace.
- Background is coset space $\mathrm{PSU}(2,2|4)/\mathrm{Sp}(2) \times \mathrm{Sp}(1,1)$.
- Isometries of background are Noether symmetries: $\mathfrak{psu}(2,2|4)$.

T-duality transformation: [Alday Maldacena][Berkovits Maldacena]

- 4 bosonic + 8 fermionic T-dualities.
- terms at worldsheet boundaries: planar!
- maps $AdS_5 \times S^5$ string model to itself: self-duality!
- maps isometries to dual isometries: dual superconformal symmetry



String Theory Integrability

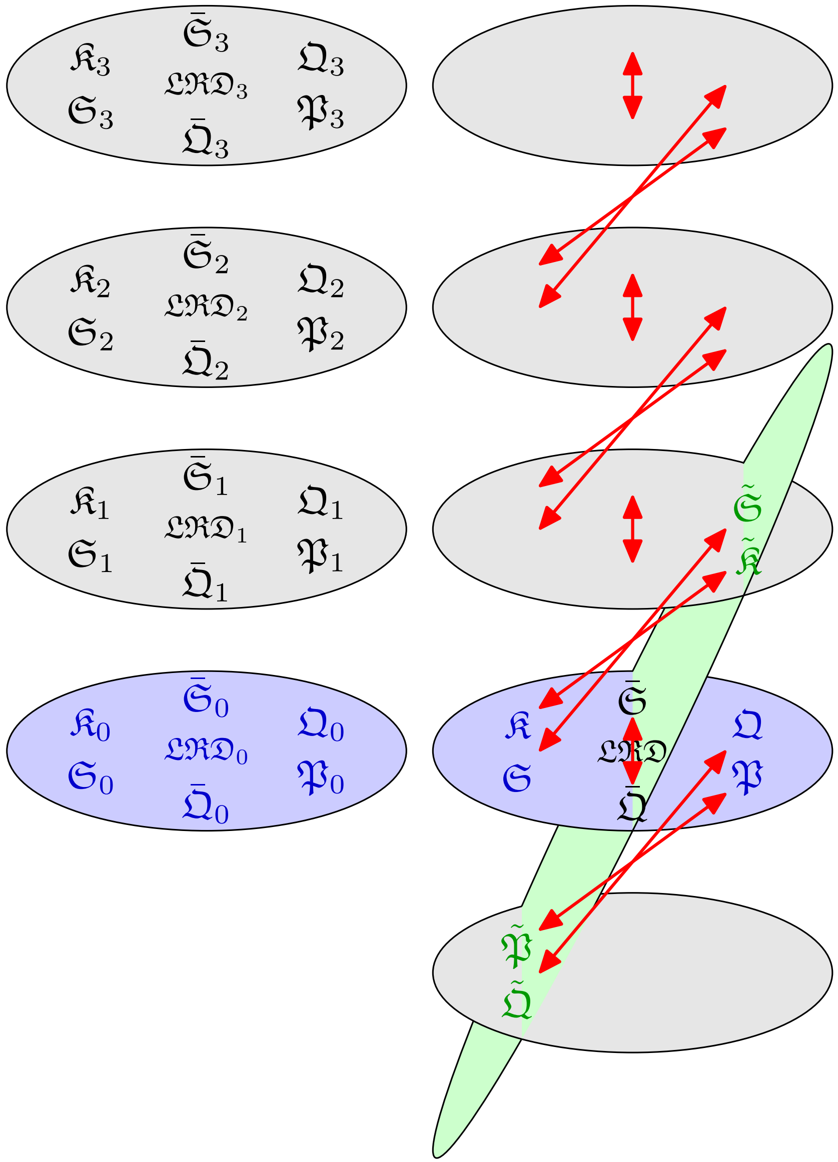
Integrability enhances conserved charges:

$$Q(z) = z Q_0 + z^2 Q_1 + z^3 Q_2 + \ldots$$

- Q_0 are Noether charges.
- Q_k form (half) loop algebra.
- ∞-dimensional hidden symmetries.

T-self-duality [NB, Ricci Tseytlin, Wolf][Berkovits Maldacena][NB 0903.0609]

- maps loop algebra to itself.
- It shows that conventional & dual superconformal symmetry close into loop algebra.
- Quantum algebra called Yangian.

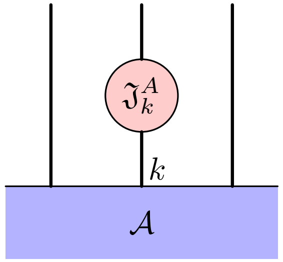


Yangian Symmetry

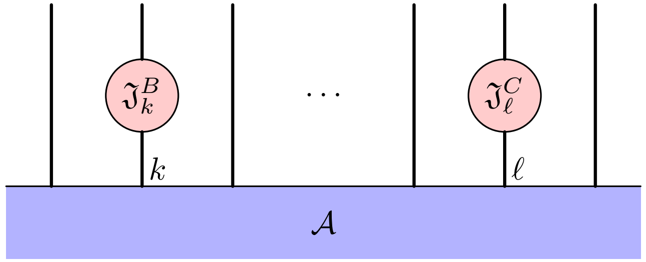
Back to scattering amplitudes in $\mathcal{N} = 4$ SYM:

Free representation $\mathfrak{J} = \mathfrak{J}_0$, $\widehat{\mathfrak{J}} = \mathfrak{J}_1$ of $\mathfrak{psu}(2,2|4)$ Yangian: [Drummond Henn Plefka]

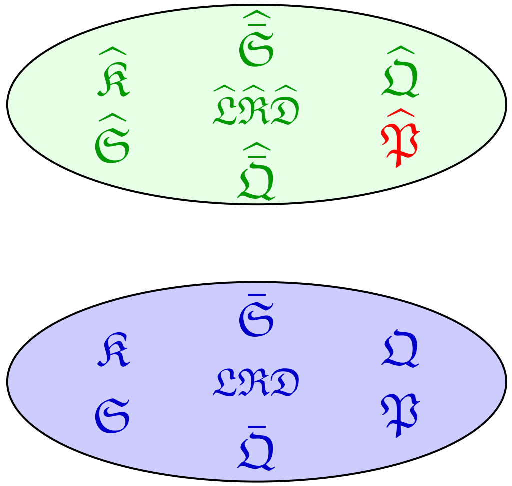
$$\mathfrak{J}^A = \sum_{k=1}^{n}$$



$$\widehat{\mathfrak{J}}^A = F^A_{BC} \sum_{k<\ell=1}^{n}$$



- Yangian Symmetry:
  Amplitudes are invariant under $\mathfrak{J}$, $\widehat{\mathfrak{J}}$. [Drummond, Henn Korchemsky Sokatchev]
- compatible with cyclic structure (exceptional)!
- $\mathfrak{J}$ & $\widehat{\mathfrak{J}}$ all one needs to know.
  In fact, only $\mathfrak{J}$ and $\widehat{\mathfrak{P}} = \tilde{\mathfrak{K}}$ sufficient.
- Representation requires planar limit & ordering;
  depends on colour structure.



Dual Conformal, Yangian, . . .

Invariant(s) & Integrability

Integrability means (pragmatic definition):

- yes we can calculate what we are interested in,
- and we can do it efficiently:
  BDS, Graßmannian, TBA, Y-system, . . . ,
- there is a hidden symmetry to constrain observables uniquely.

Tuesday/Wednesday:

learned a lot about superconformal/dual/Yangian invariants.

- Truly nice & useful results.

Not Correct!

Nitpicking

Invariants?

- "Invariants" almost (everywhere) invariant.
  Symmetries have distributional anomalies.
- Ignore at tree level $\Rightarrow$ hits you hard at loops.
  Smearing anomaly by loop integration.
- Representation requires (potentially ugly) deformations.

Invariant!

- There can be only one invariant: the S-matrix.
- S-matrix assembled from almost-invariants.

Superconformal Anomaly at Tree Level

Superconformal Boost Acting on MHV Amplitude

Apply the free superconformal generator $(\bar{\mathfrak{S}}_0)^b_{\dot\alpha} = \sum_{k=1}^n \bar\eta^b_k \bar\partial_{k,\dot\alpha}$ on

$$A_n^{\mathrm{MHV}} = \frac{\delta^4(P)\,\delta^8(Q)}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}, \qquad P^{\beta\dot\alpha} = \sum_{k=1}^n \lambda_k^\beta \bar\lambda_k^{\dot\alpha}, \quad Q^{\beta a} = \sum_{k=1}^n \lambda_k^\beta \bar\eta_k^a.$$

The anti-holomorphic derivative $\bar\partial_{\dot\alpha} = \partial/\partial\bar\lambda^{\dot\alpha}$ acts only on $\bar\lambda^{\dot\beta}$.
A_n^{MHV} is holomorphic in λ's except for $\delta^4(P)$

$$(\bar{\mathfrak{S}}_0)^b_{\dot\alpha}\delta^4(P) = \sum_{k=1}^n \eta^b_k \bar\partial_{k,\dot\alpha}\delta^4(P) = \sum_{k=1}^n \eta^b_k \lambda^\gamma_k \frac{\partial\delta^4(P)}{\partial P^{\gamma\dot\alpha}} = Q^{\gamma b}\frac{\partial\delta^4(P)}{\partial P^{\gamma\dot\alpha}}.$$

This term vanishes due to $\delta^8(Q)$. Thus $\bar{\mathfrak{S}}_0 A^{\mathrm{MHV}} = 0$. Well...

Holomorphic Anomaly

Watch out! The holomorphic anomaly yields extra contributions [Cachazo Svrcek Witten]

$$\frac{\partial}{\partial \bar{z}} \frac{1}{z} = \pi \delta^2(z), \qquad \frac{\partial}{\partial \bar{\lambda}^{\dot{\alpha}}} \frac{1}{\langle \lambda, \mu \rangle} = \pi \delta^2(\langle \lambda, \mu \rangle)\, \varepsilon_{\dot{\alpha}\dot{\beta}} \bar{\mu}^{\dot{\beta}}.$$

Taking the anomaly into account one obtains [Bargheer, NB, Galleas Loebbert, McLoughlin]

$$(\bar{\mathfrak{S}}_0)^b_{\dot{\alpha}} A_n^{\mathrm{MHV}} = -\pi \sum_{k=1}^{n} \varepsilon_{\dot{\alpha}\dot{\gamma}} \big(\bar{\lambda}^{\dot{\gamma}}_k \bar{\eta}^b_{k+1} - \bar{\lambda}^{\dot{\gamma}}_{k+1} \bar{\eta}^b_k\big) \frac{\delta^2(\langle k, k+1 \rangle)\, \delta^4(P)\, \delta^8(Q)}{\langle 12 \rangle \ldots \cancel{\langle k, k+1 \rangle} \ldots \langle n1 \rangle}.$$

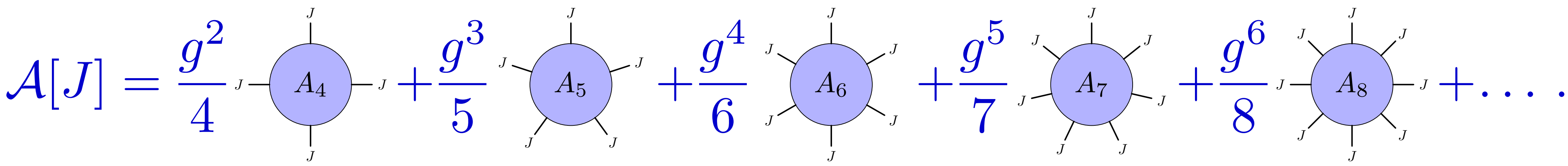
Free conformal symmetry breaks down when $\lambda_k \sim \lambda_{k+1}$: Collinear!
Note that gap $\cancel{\langle k, k+1 \rangle}$ in denominator chain of $\langle j, j+1 \rangle$ closes

$$\langle \lambda_{k-1}, \lambda_k \rangle \cancel{\langle \lambda_k, \lambda_{k+1} \rangle} \langle \lambda_{k+1}, \lambda_{k+2} \rangle \sim \langle \lambda_{k-1}, \lambda_{k,k+1} \rangle \langle \lambda_{k,k+1}, \lambda_{k+2} \rangle.$$

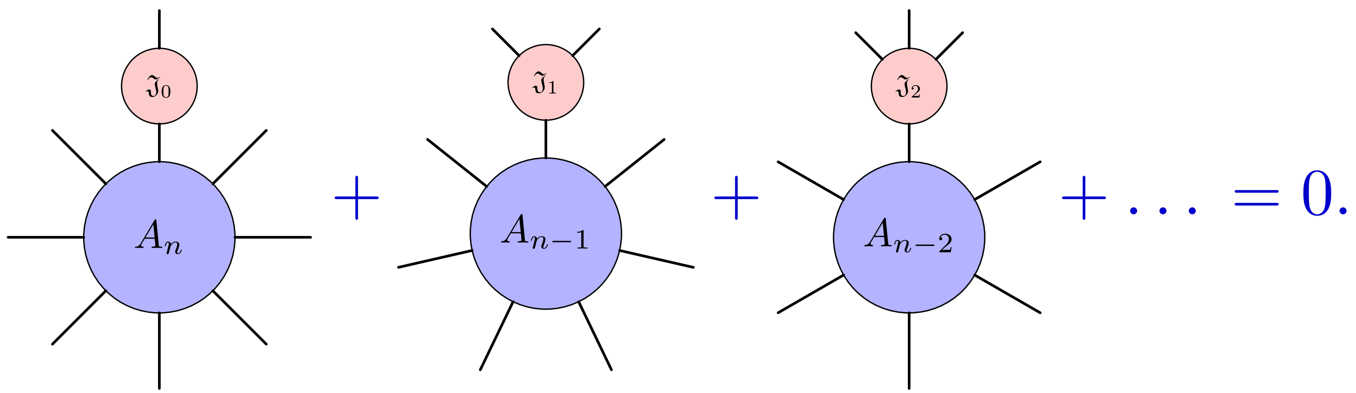
Remains $A_{n-1}^{\mathrm{MHV}}(\Lambda_1, \ldots, \Lambda_{k,k+1}, \ldots, \Lambda_n)$. Calls for $A_{n-1} \mapsto A_n$!

The S-Matrix

Package all amplitudes into generating functional with sources $J(p)$



Expected structure of symmetries at tree level



Type of action also known in classical theory as non-linear (in fields).
Example: Supersymmetry in gauge theories: $\delta\psi \sim \epsilon(D\varphi + F + [\varphi,\varphi])$.

Symmetry Correction

Formulate by acting on generating functional $(\bar{\mathfrak{S}}_0)^b_{\dot\alpha}\mathcal{A}^{\mathrm{MHV}}_n[J]$

$$\ldots = -\pi \int d^{4|4}\Lambda_{12} \prod_{k=3}^{n} d^{4|4}\Lambda_k \, d^4\bar\eta' \, d\alpha \, d\varphi \; e^{3i\varphi}\varepsilon_{\dot\alpha\dot\gamma}\bar\lambda_1^{\dot\gamma}\bar\eta_2^b$$
$$\times \operatorname{Tr}\big([J(\Lambda_1), J(\Lambda_2)]\ldots J(\Lambda_n)\big) \, A^{\mathrm{MHV}}_{n-1}(\Lambda_{12}, \Lambda_3, \ldots, \Lambda_n)$$

with the replacements

$$\lambda_1 = \lambda_{12}e^{i\varphi}\sin\alpha, \qquad \bar\eta_1 = (\bar\eta_{12}\sin\alpha + \bar\eta'\cos\alpha)e^{-i\varphi},$$
$$\lambda_2 = \lambda_{12}\cos\alpha, \qquad \bar\eta_2 = \bar\eta_{12}\cos\alpha - \bar\eta'\sin\alpha.$$

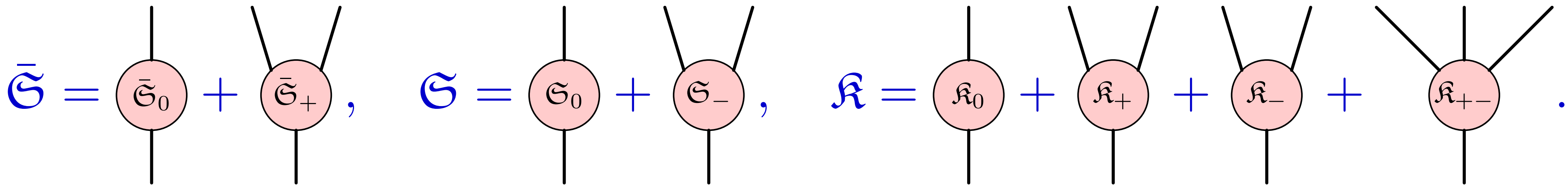
Compensate anomaly by $(\bar{\mathfrak{S}}_+)^b_{\dot\alpha}\mathcal{A}^{\mathrm{MHV}}_{n-1}[J]$ with the action on sources

$$(\bar{\mathfrak{S}}_+)^b_{\dot\alpha}J(\Lambda_{12}) = \pi \int d^4\bar\eta' \, d\alpha \, d\varphi \; e^{3i\varphi}\varepsilon_{\dot\alpha\dot\gamma}\bar\lambda_1^{\dot\gamma}\bar\eta_2^b \, [J(\Lambda_1), J(\Lambda_2)].$$

Classical Deformation for Superconformal Symmetry

Classical Representation

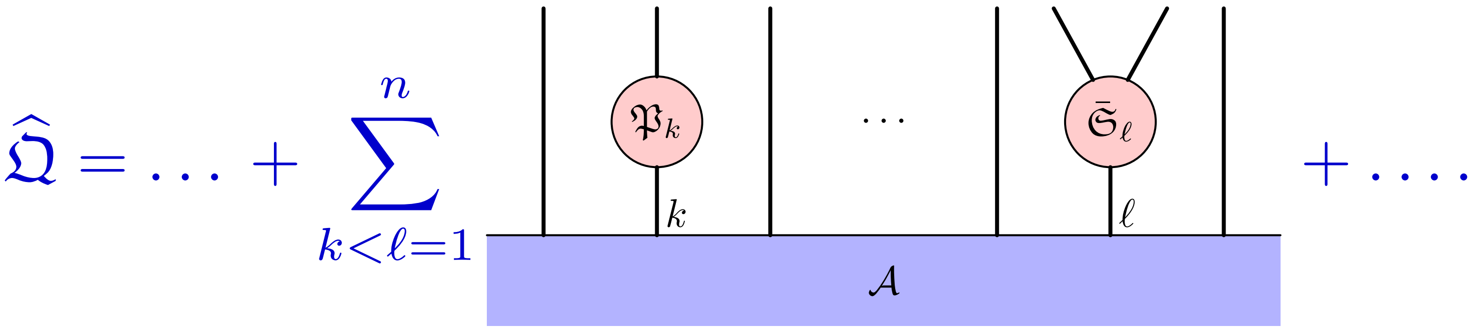
We find the following corrections for representation of $\mathfrak{S}$, $\bar{\mathfrak{S}}$ and $\mathfrak{K}$



To be done:

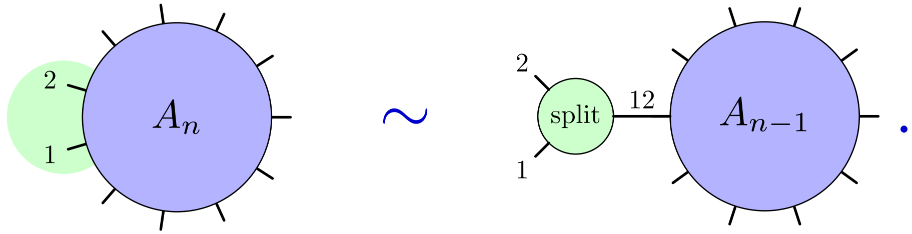
- Does the deformation annihilate all tree amplitudes?
- Is it a consistent representation of superconformal symmetry?
- What does it mean?

Similar deformations expected for classical Yangian representation, e.g.



Invariance of Tree Amplitudes

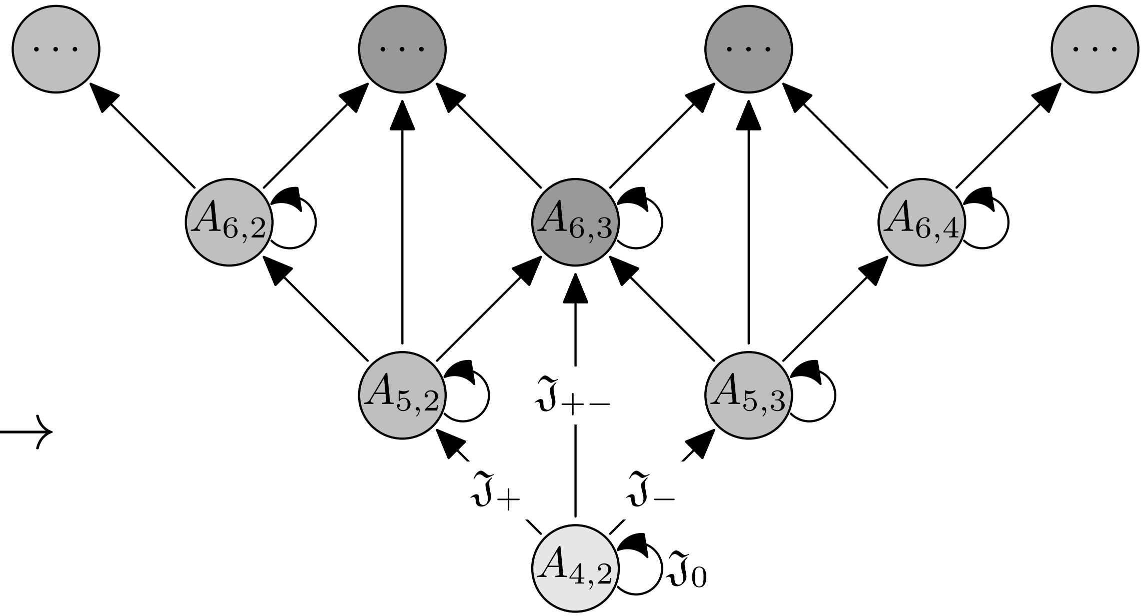
Collinear limit is universal for all (tree) amplitudes: splitting function



Follows e.g. from recursion relation by inheritance.

Exact invariance of all tree amplitudes: [Bargheer, NB, Galleas Loebbert, McLoughlin]

- Collinear singularities universal, same as for MHV.
- No anomalies from multi-particle singularities.
- Structure of cancellations $\longrightarrow$ $A_{n,k}$: n-leg N^{k-2}MHV.



Symmetry Algebra

Does the deformed representation close? [Bargheer, NB, Galleas Loebbert, McLoughlin]

- $[\mathfrak{L},\ldots]$, $[\mathfrak{R},\ldots]$: proper index contractions.
- $[\mathfrak{D},\ldots]$: proper conformal weights.
- $\{\mathfrak{Q},\mathfrak{S}\}$, $\{\mathfrak{Q},\bar{\mathfrak{S}}\}$, $\{\bar{\mathfrak{Q}},\mathfrak{S}\}$, $\{\bar{\mathfrak{Q}},\bar{\mathfrak{S}}\}$: straight-forward.
- $\{\mathfrak{S},\mathfrak{S}\}$, $\{\bar{\mathfrak{S}},\bar{\mathfrak{S}}\}$: rather non-trivial, requires vanishing central charge. Closes only onto gauge transformation

$$\{\mathfrak{S}_{\alpha b},\mathfrak{S}_{\gamma d}\}=\varepsilon_{\alpha\gamma}\mathfrak{G}_{bd},\qquad \{\bar{\mathfrak{S}}^{b}_{\dot\alpha},\bar{\mathfrak{S}}^{d}_{\dot\gamma}\}=\tfrac{1}{2}\varepsilon_{\dot\alpha\dot\gamma}\varepsilon^{bdef}\mathfrak{G}_{ef},\qquad \mathfrak{G}_{ab}J(\Lambda)\sim[\partial_a\partial_b J(0),J(\Lambda)].$$

  On gauge invariant objects effectively $\{\mathfrak{S},\mathfrak{S}\}=0=\{\bar{\mathfrak{S}},\bar{\mathfrak{S}}\}$.
- $\{\mathfrak{S}_{\alpha b},\bar{\mathfrak{S}}^{d}_{\dot\gamma}\}=\delta^d_b\mathfrak{K}_{\alpha\dot\gamma}$: defines conformal boost $\mathfrak{K}$.
- $[\mathfrak{K},\ldots]$, $[\mathfrak{P},\ldots]$: through Jacobi identities.

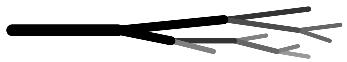
Superconformal algebra satisfied, but contains gauge transformations.

Implications

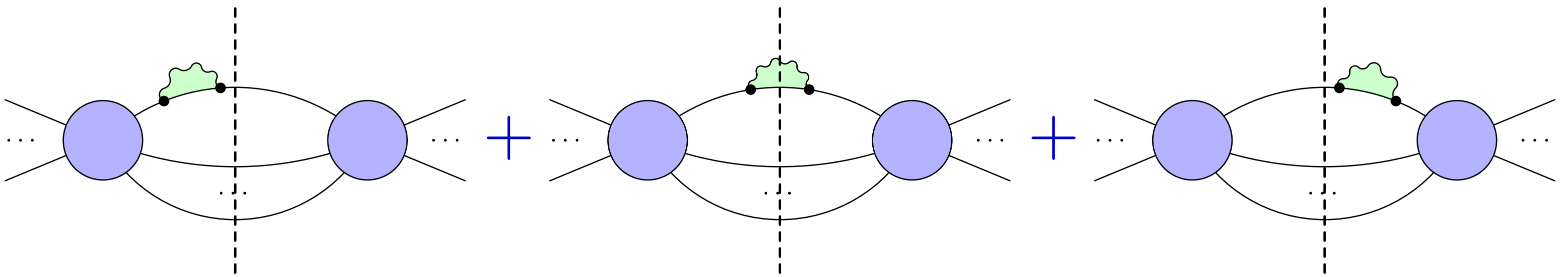
Massless Asymptotic States

Reconsider conceptual problems of massless scattering:

- No mass gap: Massless particles can "decay" into particle showers



- Particle number not well-defined in asymptotic region.
- But: Shower particles are strictly collinear.
  Single massless particle physically indistinguishable from shower.
- Overcounting of collinear states leads to IR divergencies at loop level.
- Can cancel IR divergencies in cross sections:



Finite, but scattering amplitudes are more convenient.

Symmetry for Massless Asymptotic States

Asymptotic space:

- Fock space is too large; overcounts collinear states.
- Should project out collinear states: Conceptually hard!
- Rather embed asymptotic space in larger Fock space.
  Keep collinear issues in mind.

Our results are in line with the above:

- Conformal anomaly precisely where asymptotic particles overcount.
- Deformation makes superconformal representation
  compatible with embedding of asymptotic space into Fock space. (?)
- Exact conformal invariance incompatible with fixed particle number.
  Have to consider generating functional $\mathcal{A}$ instead of amplitude(s) A.
  Scattering operator for asymptotic space instead of scattering matrix.

Uniqueness

All tree amplitudes have been constructed by recursion relations [Drummond Henn]

$$A_{n,k}(p) = A_n^{\mathrm{MHV}}(p) \sum_\alpha c_\alpha R_\alpha(p).$$

Each R is (almost) invariant under the free Yangian representation.
How to obtain the correct physical linear combination $c_\alpha = 1$?

- Demand absense of spurious singularities or equivalently [Hodges 0905.1473]
- demand correct collinear behaviour. [Korchemsky Sokatchev]

Deformed representation ensures correct collinear behaviour. [Bargheer, NB, Galleas Loebbert, McLoughlin]
Therefore symmetry alone fixes correct linear combination $c_\alpha = 1$!

Very important for construction by symmetry at higher loops:

- Adding any invariant respects symmetry: ambiguity!
- Adding tree level amounts to an overall factor; okay.

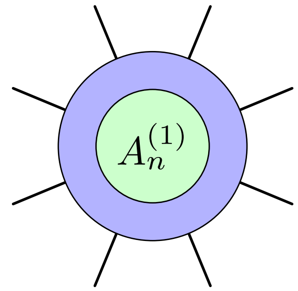
Can symmetry fix planar amplitude completely (non)perturbatively?

Superconformal Symmetry of Loop Amplitudes

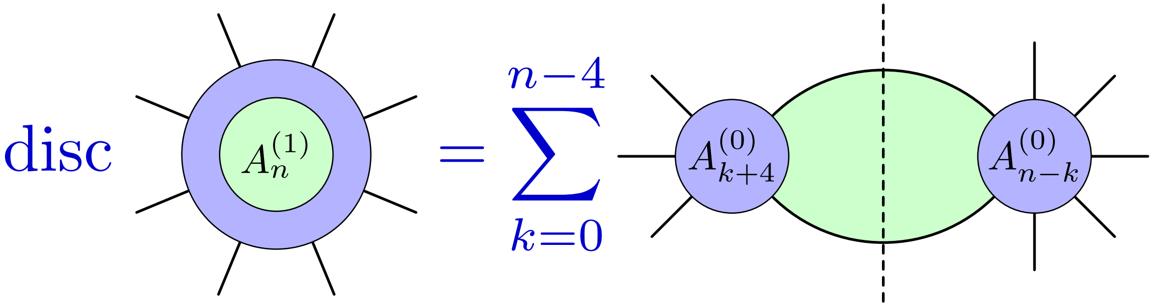
Symmetry of Unitarity Cuts

What about conformal symmetry for loop amplitudes?

- Particles in loop are off-shell.
- Conformal symmetry applies to on-shell particles only.
- How to determine loop anomaly?



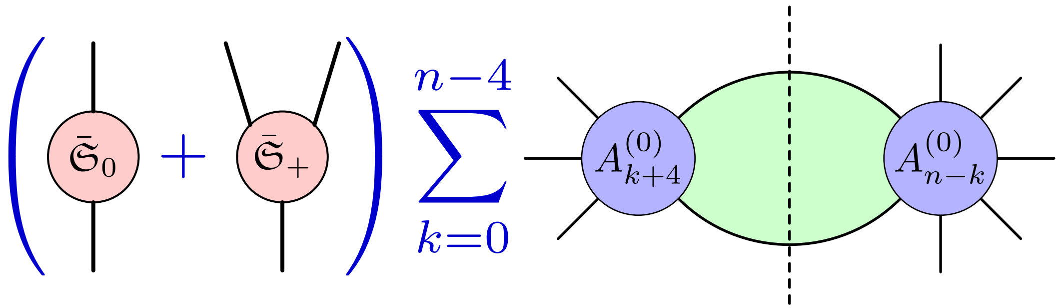
Idea: Consider unitarity cut of loop amplitude [Korchemsky Sokatchev] [Brandhuber Heslop Travaglini] [Sever Vieira]



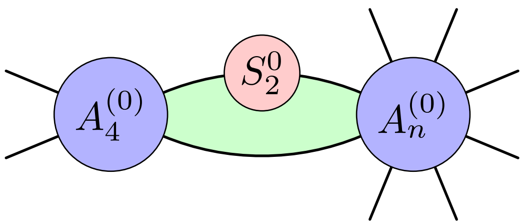
- External and loop particles on shell!
- Only tree amplitudes used.
- Can act with symmetry to determine anomaly of cut.
- Use dispersion integral to reconstruct anomaly of loop later.

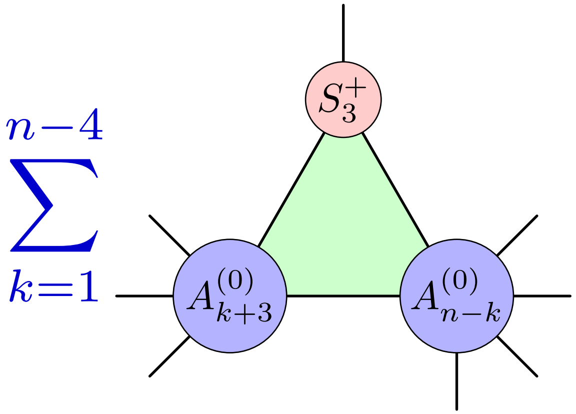
Anomaly of Unitarity Cuts

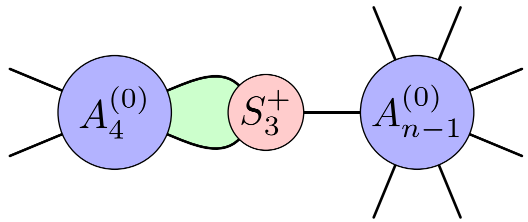
Act with superconformal boost on unitarity cut using invariance of trees



Three remaining contributions: [NB, Henn McLoughlin, Plefka]







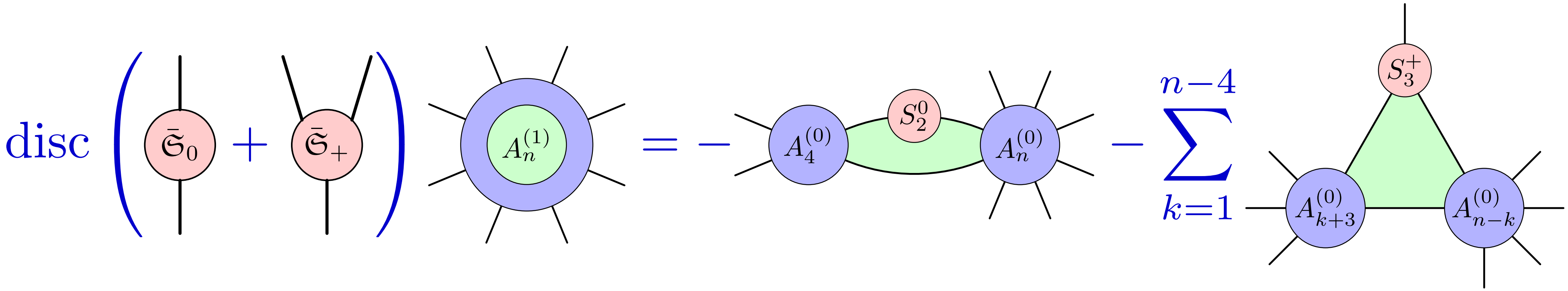
- from divergent measure
- no momentum transfer

- integral localised
- finite, rational

- one-loop splitting
- ill-defined→zero

One-Loop Anomaly

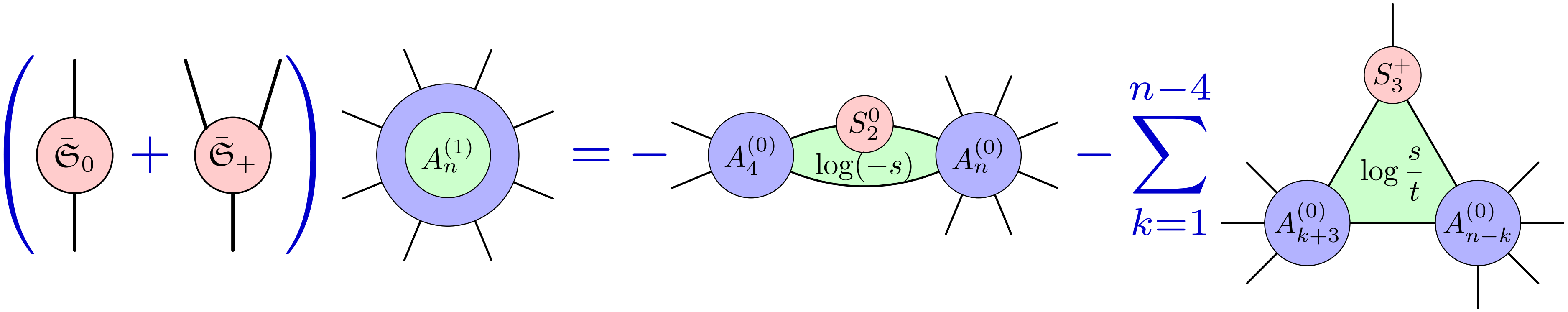
The anomaly of unitarity cuts reads



How to promote it to the loop anomaly?

[NB, Henn McLoughlin, Plefka]

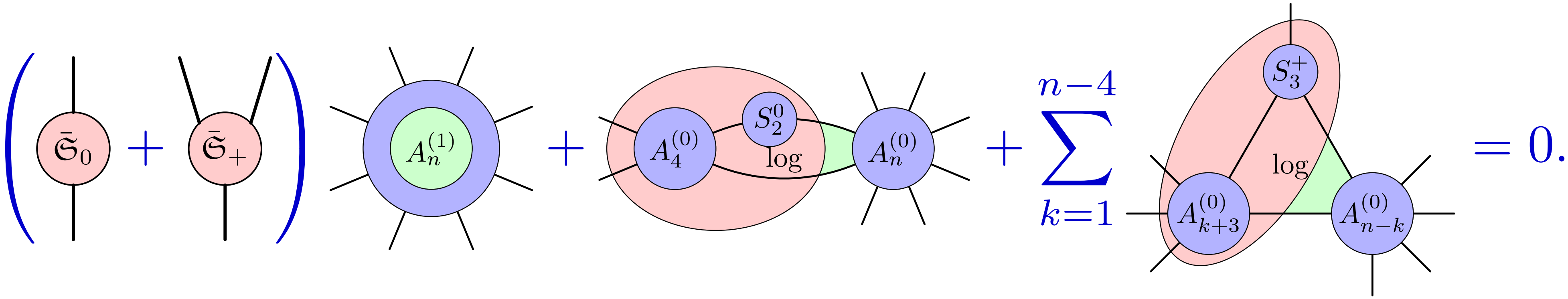
- All integrals are fully localised, cut anomaly is rational.
- Multiply by logs of cut invariants to reproduce cut discontinuity.



Confirmed explicitly for MHV and 6pt NMHV amplitudes.

One-Loop Invariance

Can rewrite loop anomaly as invariance statement: [NB, Henn McLoughlin, Plefka]



One-loop superconformal invariance of scattering amplitudes

$$\bar{\mathfrak{S}}^{(0)}\mathcal{A}^{(1)} + \bar{\mathfrak{S}}^{(1)}\mathcal{A}^{(0)} = 0.$$

- Different from earlier proposal: [Sever Vieira]
  Uses only on-shell amplitudes, compatible with regulator.
- Tree anomalies: collinearities $(1 \to 2)$.
- Loop anomalies: measure $(2 \to 2)$, loop collinearities $(2 \to k)$.

Superconformal Algebra

Do the loop corrections of the algebra close?

- Measure contributions take a simple universal form $\Rightarrow$ closure

$$\mathfrak{J}^{(1)}_{2\to 2} \sim [X_{2\to 2}, \mathfrak{J}^{(0)}], \qquad X_{2\to 2} = \sum_{k=1}^{n} \frac{1}{\epsilon^2} \left(\frac{s_{k,k+1}}{-\mu^2} \right)^{-\epsilon} .$$

- Corrections $\bar{\mathfrak{S}}^{(1)}_{2\to k}$ are invariant under super-Poincaré.

Rest of superconformal algebra is not yet clear:

- Some indications that algebra may not close properly.
- Deformations for $\bar{\mathfrak{S}}^{(1)}_{2\to k}$ not uniquely determined by amplitudes.
- Closure on which class of functions?
  Only amplitude itself: only singlet representation consistent?
- Can superconformal algebra be defined at loop level?

In any case, we understand the one-loop superconformal anomaly.

Integrability
of Loop Amplitudes

Yangian Anomaly

What about integrability at one loop?

One-loop (exact) anomaly of dual conformal generator $\widetilde{\mathfrak{K}}$

[Drummond, Henn, Korchemsky, Sokatchev] [Drummond, Henn, Korchemsky, Sokatchev] [Brandhuber, Heslop, Travaglini] [Elvang, Freedman, Kiermaier] [Korchemsky, Sokatchev] [Brandhuber, Heslop, Travaglini]

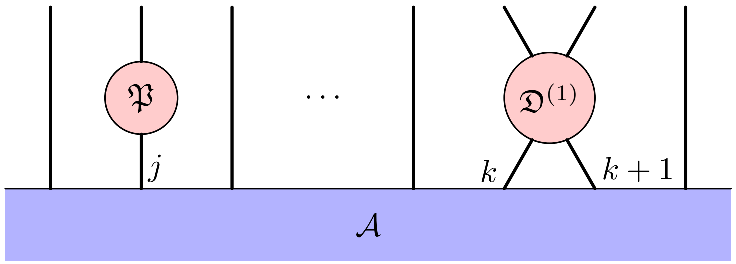
$$\widetilde{\mathfrak{K}}^{\beta\dot\alpha} A^{(1)} \sim \sum_{k=1}^{n} \frac{1}{\epsilon} \left(\frac{s_{k,k+1}}{-\mu^2} \right)^{-\epsilon} \sum_{j=1}^{k} p_j^{\beta\dot\alpha} A^{(0)}.$$

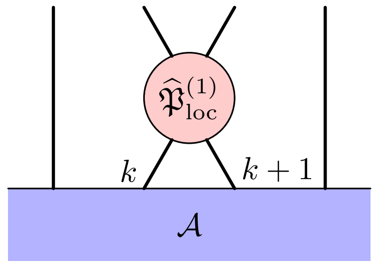
Dual conformal generator is bi-local momentum generator $\widetilde{\mathfrak{K}} \simeq \widehat{\mathfrak{P}}$

$$\widehat{\mathfrak{P}} \simeq \mathfrak{P} \wedge \mathfrak{D} + \mathfrak{P} \wedge \mathfrak{L} + \mathfrak{Q} \wedge \bar{\mathfrak{Q}}.$$

One-loop correction compensates the above anomaly. [NB, Henn, McLoughlin, Plefka]

$$\widehat{\mathfrak{P}}^{(1)} \simeq \mathfrak{P} \wedge \mathfrak{D}^{(1)}_{2\to 2} + \widehat{\mathfrak{P}}^{(1)}_{\mathrm{local}}$$

$$\simeq \textstyle\sum_{j<k=1}^{n}$$ 

$$+ \textstyle\sum_{k=1}^{n}$$ 

Dual Superconformal Anomaly

Loop anomaly of dual superconformal symmetry $\widetilde{\mathfrak{S}}$ complicated. [Korchemsky Sokatchev]

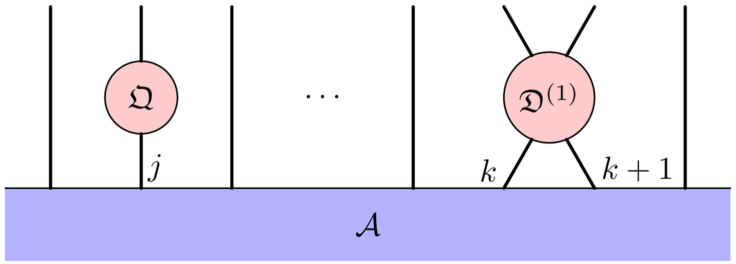
Generic form of the Yangian generator $\widetilde{\mathfrak{S}} = \widehat{\mathfrak{Q}}$

$$\widehat{\mathfrak{Q}} \simeq \mathfrak{P} \wedge \bar{\mathfrak{S}} + \mathfrak{Q} \wedge \mathfrak{L} + \mathfrak{Q} \wedge \mathfrak{R} + \mathfrak{Q} \wedge \mathfrak{D}$$

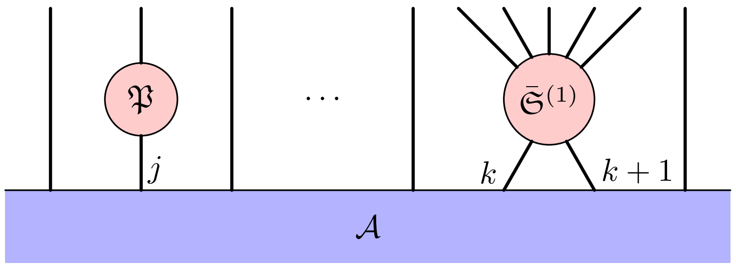
Follows from commutator $\widehat{\mathfrak{Q}} = [\bar{\mathfrak{S}}, \widehat{\mathfrak{P}}]$ with anomalous $\bar{\mathfrak{S}}$ [NB, Henn McLoughlin, Plefka]

$$\widehat{\mathfrak{Q}}^{(1)} \simeq \mathfrak{P} \wedge \bar{\mathfrak{S}}^{(1)}_{2\to k} + \mathfrak{Q} \wedge \mathfrak{D}^{(1)}_{2\to 2} + \widehat{\mathfrak{Q}}^{(1)}_{\text{local}}$$

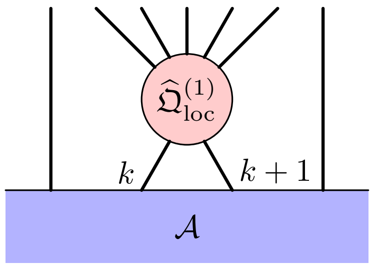
$\simeq \sum_{j<k}$



$+ \sum_{j<k}$



$+ \sum_{k}$



Conclusions

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⋆ **Superconformal Symmetry at Tree Level**

- Tree amplitudes almost invariant under free superconformal symmetry.
- Invariance violated for singular configurations: Collinear momenta.
- Transformations can be corrected to make trees fully invariant.
- Dynamic corrections requires, changes number of legs.
- Superconformal algebra closes onto gauge transformations.
- Yangian appears to lead to a unique invariant $\Rightarrow$ the S-matrix.

⋆ **Superconformal Symmetry at One Loop**

- Transformations can be corrected to make loops fully invariant.

⋆ **Open Problems**

- How does the algebra at loop level close?
- What about conformal inversions?