Twistor String and Grassmannian: All Trees in $\mathcal{N} = 4$ Yang-Mills

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Spradlin, AV, arXiv:0909.0229

Nandan, Wen, AV, arXiv:0912.3705

Bourjaily, Trnka, AV, Wen, to appear

Introduction

- There is now a vast and growing body of evidence to support the conjectured duality between scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills and certain contour integrals over the Grassmannian [ACCK]

- In Witten's twistor string theory tree amplitudes are computed via the [RSV] connected prescription as integrals of an open string correlator over the moduli space of curves in supertwistor space

- In my talk, I will explain how Grassmannian integral can be derived from the connected prescription integral by a simple integral deformation

- The twistor string theory connected prescription will indicate which particular contour should be taken in the Grassmannian to compute all tree level amplitudes

Outline

- **Twistor String Theory: linking the connected prescription**

[Witten][Roiban, Spradlin, AV][Spradlin, AV] [Dolan, Goddard]

- **Grassmannian formulation of $\mathcal{N} = 4$ Yang-Mills**

[Arkani-Hamed, Cachazo, Cheung, Kaplan]

- **The relation between them: contours and integrands**

[Spradlin, AV] [Dolan, Goddard]^2 [Nandan, AV, Wen] [Arkani-Hamed, Bourjaily, Cachazo, Trnka]^2 [Bourjaily, Trnka, AV, Wen]

A Review of Twistor String Theory

In the beginning, there was Twistor String Theory [Witten (2003)].

Nair (1988) observed that MHV amplitudes (the Parke-Taylor formula) could be written as the integral of a certain WZW current algebra correlator:

$$\mathcal{A}^{\mathrm{MHV}}(\lambda_i, \widetilde{\lambda}_i) = \int d^4x \exp\left(i x_{a\dot{a}} \lambda_i^a \widetilde{\lambda}_i^{\dot{a}}\right) \prod_{i=1}^{n} \frac{1}{\langle i\, i+1 \rangle}$$

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Then, recognizing d^4x as the measure on the moduli space of lines in $\mathbb{P}^3$, Witten (2003) checked via difficult calculation several cases of his conjecture that:

The N^{k}MHV superamplitude in super Yang-Mills is supported on curves in $\mathbb{P}^{3|4}$ of degree $k+1$.

The ‘Connected Prescription’

According to twistor string theory, the N^{k}MHV superamplitude is given explicitly by [Roiban, Spradlin, AV (2004)]:

$$\mathcal{A}^{\mathrm{N}^k\mathrm{MHV}}(\mathcal{Z}_i) = \int [\mathcal{D}\mathcal{C}_{k+1}] \, \frac{d^n z}{(z_1 - z_2)\cdots(z_n - z_1)} \prod_{i=1}^{n} \delta^{3|4}(\mathcal{Z}_i - \mathcal{C}_d(z_i))$$

- $\mathcal{Z}_i = (\lambda_i, \mu_i)$ where μ_i is related to $\widetilde{\lambda}_i$ by “Fourier transform”.
- $\mathcal{C}_d(z)$ denotes a degree d curve in $\mathbb{P}^{3|4}$.
- $[\mathcal{D}\mathcal{C}_d]$ denotes the measure on the moduli space of such curves
- $\prod \frac{1}{z_i - z_{i+1}}$ is the WZW current algebra correlator—here arising from vertex operators of open strings ending on an instanton which wraps the curve $\mathcal{C}(z)$.
- The delta functions force the specified $\mathcal{Z}_i$ to lie on the curve!

Properties of the Connected Prescription

- Conformal and dihedral $i \to i+1, i \to n+1-i$ symmetries are manifest for all superamplitudes
- Parity symmetry is almost manifest [RSV, Witten]
- Possesses the correct collinear and soft limits
- A little processing reveals that the formula must be interpreted as a contour integral of the form

$$\int d^m z \frac{h(\vec{z})}{f_1(\vec{z}) \cdots f_m(\vec{z})} = \sum_{\vec{z}_* : f_1(\vec{z}_*) = \cdots = f_m(\vec{z}_*) = 0} h(\vec{z}_*) \left[\det \left(\frac{\partial f_i}{\partial z_j} \right) \right]^{-1}_{\vec{z} = \vec{z}_*}$$

- So, calculating any superamplitude reduces to the problem of finding the roots of some polynomial equations.

The numbers of roots are Eulerian numbers:

n=4	1						
n=5	1	1					
n=6	1	4	1				
n=7	1	11	11	1			
n=8	1	26	66	26	1		
n=9	1	57	302	302	57	1	
n=10	1	120	1191	2416	1191	120	1
	MHV	NMHV	NNMHV				

The total number of roots is $(n-3)!$ — so the formula is conceptually beautiful, but sadly computationally useless!

Fourier Transforming Connected Prescription

Motivated by [Arkani-Hamed, Cachazo, Cheung & Kaplan] [Mason, Skinner], let us consider "Fourier transforming" some of the twistor variables

$$\mathcal{Z}_i = (\lambda_i, \mu_i) \qquad \rightarrow \qquad \mathcal{W}_i = (\widetilde{\mu}_i, \widetilde{\lambda}_i)$$

via

$$\mathcal{A}(\mathcal{W}_a, \mathcal{Z}_J) = \int \exp\left(i \sum_a \mathcal{W}_a \cdot \mathcal{Z}_a \right) \mathcal{A}(\mathcal{Z}_i)$$

For N^{k}MHV superamplitude, an astoundingly convenient choice is to leave precisely $k+2$ particles in the $\mathcal{Z}$ representation and transform all others to $\mathcal{W}$.

The integral over the moduli space of curves is then a triviality...

The ‘Connected Link’ Formula

... leading to the formula

$$\mathcal{A}(\mathcal{W}_i, \mathcal{Z}_J) = \int d^{(n-k-2)\times(k+2)} c_{iJ}\, U(c_{iJ})\ \exp\left(i \sum_{i,J} c_{iJ} \mathcal{W}_i \cdot \mathcal{Z}_J \right)$$

with the ‘link representation’

$$U(c_{iJ}) = \int \frac{d^n z}{(z_1 - z_2)\cdots(z_n - z_1)} \frac{d^n c}{c_1 \cdots c_n} \prod_{i,J} \delta\left(c_{iJ} - \frac{c_i c_J}{z_i - z_J} \right)$$

Reminder: this is a contour integral, with the delta-functions indicating which singularities the contour is supposed to encircle.

An Example: $A(1^+, 2^-, 3^+, 4^-, 5^+, 6^-)$

The link representations are simple to work out on a case-by-case basis, for example

$$U^{+-+-+-} = \frac{1}{c_{14}c_{36}c_{52}}\delta(S_{135:246})$$

Generically, the N^{k}MHV superamplitude involves $k(n-k-4)$ delta-functions of 'sextics':

$$S_{ijk:lmn} = c_{im}c_{in}c_{jl}c_{kl}c_{km}c_{jn} \pm 5 \text{ permutations}$$

Returning to Physical Space

Now the biggest benefit of 'link representations' is that going to physical space is trivial:

$$A(\lambda_i, \widetilde{\lambda}_i) = \int d^{(n-k-2)\times(k+2)} c_{iJ}\, U(c_{iJ}) \prod_i \delta^2(\lambda_i - c_{iJ}\lambda_J) \prod_J \delta^2(\widetilde{\lambda}_J + c_{iJ}\widetilde{\lambda}_i)$$

The delta-functions here give $2n-4$ linear equations in terms of $(n-k-2)(k+2)$ variables, which can be solved in terms of $k(n-k-4)$ parameters (we'll call them τ).

Returning to our example we now have

$$A^{+-+-+-} = \int d\tau \frac{1}{c_{14}c_{36}c_{52}} \delta(S_{135:246})$$

where c is linear in τ and the $S_{ijk:lmn}$ are quartic.

By choosing numerical values for the external kinematics and summing over four roots of $S_{135:246}$ one can verify that this integral reproduces the correct amplitude.

This is just the familiar [RSV] connected prescription, in new variables.

A Contour Integral

But consider more generally the object

$$T^{+-+-+-}(\tau) = \frac{1}{c_{14}c_{36}c_{52}} \frac{1}{S_{135:246}}$$

Apparently we've learned that the contour integral

$$\oint d\tau \, T^{+-+-+-}(\tau)$$

over the contour which encircles the four poles of the sextic calculates the 'connected prescription representation' for the tree-level amplitude.

But T^{+-+-+-} has three other poles in the τ plane. What do those residues compute?

By Cauchy's theorem we know that the sum of these three residues computes minus the amplitude,

$$A^{+-+-+-} = -\int d\tau \frac{1}{S_{135:246}} \delta(c_{14} c_{36} c_{52})$$

It is simple to calculate the corresponding residues analytically, and one obtains BCFW representation for the amplitude

$$A^{+-+-+-} = \frac{\langle 1\,3\rangle^4 [4\,6]^4}{\langle 1\,2\rangle\langle 2\,3\rangle[4\,5][5\,6] s_{123} [6|1+2|3\rangle[4|3+2|1\rangle} + (i \to i+2) + (i \to i+4)$$

The twistor string connected prescription gives BCFW representation for the amplitude! [Spradlin, AV]

Rewrite in terms of minors

$$T^{+-+-+-}(\tau) = \frac{1}{c_{14}c_{36}c_{52}} \frac{1}{S_{135:246}}$$

$$C = \begin{pmatrix} 1 & c_{12} & 0 & c_{14} & 0 & c_{16} \\ 0 & c_{32} & 1 & c_{34} & 0 & c_{36} \\ 0 & c_{52} & 0 & c_{54} & 1 & c_{56} \end{pmatrix}$$

$$(123) = c_{52} \quad (345) = c_{14} \quad (561) = c_{36} \quad \text{etc}$$

$$T_{6;3} = \int d\tau \frac{(135)}{(123)(345)(561)} \frac{1}{[(234)(456)(612)(135) - (123)(345)(561)(246)]}$$

This is very similar to the Grassmanian formula!

Grassmannian formulation of $\mathcal{N}=4$ Yang-Mills

Arkani-Hamed, Cachazo, Cheung, Kaplan conjectured that that the leading singularities of n-particle $\mathrm{N}^{\mathrm{k}-2}\mathrm{MHV}$ amplitudes are captured by the contour integral over the Grassmannian which may be parametrized by a $k \times n$ matrix $C_{\alpha\, a}$

$$\boxed{\mathcal{L}_{n,k} = \frac{1}{\mathrm{vol}[\mathrm{GL}(k)]} \oint_{\Gamma_{n,k}} \frac{d^{k\times n} C_{\alpha\, a}}{(1)(2)(3)\cdots(n-1)(n)} \prod_{\alpha=1}^{k} \delta^{4|4}\left(C_{\alpha\, a} \mathcal{W}_a\right)}$$

$\mathcal{W}_a \equiv (\tilde{\mu}, \tilde{\lambda}|\tilde{\eta})_a$, for $a \in \{1, \ldots, n\}$ are supertwistors which encode the external momenta and helicity data;

(j) represents the j^{th} $k \times k$-minor of $C_{\alpha\, a}$ built out of consecutive columns

$$(j) \equiv (j\ \ j+1\ \ \cdots\ \ j+k-1) \equiv \epsilon^{\alpha_1\, \alpha_2 \cdots \alpha_k} C_{\alpha_1\, j} C_{\alpha_2\, j+1} \cdots C_{\alpha_k\, j+k-1}.$$

Grassmannian formulation of $\mathcal{N} = 4$ Yang-Mills

- The different choices of contour compute BCFW representations of tree-level amplitudes as well as leading singularities of loop amplitudes to all orders.

- The formula has manifest cyclic, parity, superconformal and dual superconformal symmetries.

- Lots of progress

[Arkani-Hamed, Cachazo, Cheung, Kaplan]^2 [Mason, Skinner] [Bullimore, Mason, Skinner] [Drummon, Ferro][Arkani-Hamed, Bourjaily, Cachazo, Trnka]^2 [Korchemsky, Sokatchev][Dolan, Goddard]^2 [Spradlin AV][Nandan, Wen, AV][Hodges]

- How does one choose a contour?

Example: Six Points Contour [ACCK]

$$\mathcal{L}_{6;3} = \int \frac{d\tau}{[(123)(234)(345)(456)(561)(612)](\tau)}$$

- Choose a convenient $GL(3)$ gauge fixing

$$C = \begin{pmatrix} c_{21} & 1 & c_{23} & 0 & c_{25} & 0 \\ c_{41} & 0 & c_{43} & 1 & c_{45} & 0 \\ c_{61} & 0 & c_{63} & 0 & c_{65} & 1 \end{pmatrix}$$

- Solve delta functions

$$\lambda_i - c_{Ii}\lambda_I = 0 \quad \tilde{\lambda}_I + c_{Ii}\tilde{\lambda}_i = 0$$

$$c_{Ii} = c^*_{Ii} + \epsilon_{ijk}\epsilon_{IJK}[jk]\langle JK\rangle\tau \quad \text{where} \quad c^*_{21} = \frac{[23]}{[12]}, \quad c^*_{41} = \frac{\langle 6|5+4|3]}{\langle 46\rangle[13]}, \text{etc}$$

- Each of the minors is linear in τ

- Compute residues at the poles

$$(1) = -\frac{[3|(2+4)|6\rangle^4}{[23][34]\langle 56\rangle\langle 61\rangle(p_5+p_6+p_1)^2\langle 1|6+5|4]\langle 5|6+1|2]},$$

$$(4) = \frac{\langle 46\rangle^4[13]^4}{[12][23]\langle 45\rangle\langle 56\rangle(p_4+p_5+p_6)^2\langle 6|5+4|3]\langle 4|5+6|1]},$$

$$(3) = g^2(1),\ (5) = g^4(1),\ (6) = g^2(4), (2) = g^4(4)$$

- Recognize that particular sums of these residues give BCFW and P(BCFW) terms in the amplitude.

$$M_{BCFW}^{+-+-+-} = (234) + (456) + (612)$$

$$M_{P(BCFW)}^{+-+-+-} = -(123) - (345) - (561)$$

- One identifies the tree amplitude with a particular choice of contour. Depending on what contour is chosen one gets different forms of amplitude.

Summary: Six Points

- Connected formula

$$T_{6;3} = \int d\tau \frac{(135)}{(123)(345)(561)} \frac{1}{[(234)(456)(612)(135) - (123)(345)(561)(246)]}$$

- Cauchy's theorem relates connected formula to ACCK formula

$$\{\text{Sextic}\} = -(123) - (345) - (561)$$

- We can deform the sextics by a non-zero parameter t

$$\text{Sextic} \to (234)(456)(612)(135) - t(123)(345)(561)(246)$$

- $t = 0$ is Grassmannian and $t = 1$ is connected prescription

Global Residue Theorem

- To explore higher point amplitudes we can use Global Residue Theorem:

$$\oint_{f_1=\cdots=f_n=0} d^n z \, \frac{h(z)}{f_1(z)\cdots f_n(z)} = 0$$

where $h(z)$ is a polynomial of degree less than $\sum \deg f_i - (n+1)$, so that it has no poles at finite z and the integrand falls off sufficiently fast to avoid poles at infinity.

- Checked that the connected prescription gives Yang-Mills amplitude in BCFW form: six and seven particles [Spradlin, AV] [Dolan, Goddard],all NMHV amplitudes [Nandan, AV, Wen], split helicity [Dolan, Goddard]

ACCK Contour for NMHV

- ACCK showed that the BCFW contour for NMHV amplitude is given by products of strictly increasing $n-5$ residues

$$\Gamma^{\mathcal{L}}_{n;3} = \sum (\mathrm{Odd})(\mathrm{Even})(\mathrm{Odd})(\mathrm{Even})\ldots.$$

- For example, seven-point NMHV amplitude can be written as an integral over the contour

$$\Gamma^{\mathcal{L}}_{7;3} = (2)[(3)+(5)+(7)] + (4)[(5)+(7)] + (6)(7)$$

Example of using GRT: Seven Points

- Twistor string connected prescription [Nandan, Wen, AV]

$$T_{7;3} = \int d^2\tau \frac{(135)(235)(612)(136)}{(1)(3)(6)} \frac{1}{S_1 S_2}$$

$$S_1 = (2)(4)(612)(135) - (1)(3)(561)(246)$$

$$S_2 = (5)(7)(235)(136) - (1)(3)(6)(572)$$

- Applying global residue theorem we get

$$\{S_1, S_2\} =$$

$$= \{1, S_1\} + \{3, S_1\} + \{6, S_1\}$$

$$= \{1,2\} + \{1,4\} + \cancel{\{3,2\}} + \{3,4\} + \{6, S_1\}$$

$$= \{1,2\} + \{1,4\} + \{3,4\} - \{6, S_2\} - \{6,1\} - \{6,3\}$$

$$= \{1,2\} + \{1,4\} + \{3,4\} - \{6,5\} - \cancel{\{6,7\}} - \{6,1\} - \{6,3\}$$

- Non-adjacent minors are canceled by the numerator
- $\{3, 2\} = 0$ implies $(235) = 0$, which is canceled by the numerator
- To simplify the residue $\{6, S_1\}$ use GRT again
- Collecting all the residues we get

$$\{S_1, S_2\} = (1)[(2) + (4) + (6)] + (3)[(4) + (6)] + (5)(6)$$

These are exactly the contours giving BCFW representation

- We arrived to Grassmannian from connected twistor string expression via Global Residue Theorem.
- Similar calculation works for n-point NMHV

Deformation of the integrand

- The above proof does not depend on parameters t_1 and t_2

$$T_{7;3} = \int d^2\tau \frac{(135)(235)(612)(136)}{(1)(3)(6)} \frac{1}{S_1 S_2}$$

$$S_1 = (2)(4)(612)(135) - t_1(1)(3)(561)(246)$$

$$S_2 = (5)(7)(235)(136) - t_2(1)(3)(572)(671)$$

- $t_1 = t_2 = 1$: twistor string connected prescription

- $t_1 = t_2 = 0$: Grassmanian unified as a single variety

[Nandan, Wen, AV] [Arkani-Hamed, Bourjaily, Cachazo, Trnka]

Unifying Contour as a Variety

- One can unify the residues into a single variety for NMHV and N^2MHV [Arkani-Hamed, Bourjaily, Cachazo, Trnka]

$$A_n^{(3)} = \int_{\mathbf{f_n=0}} d^{n-5}\tau \frac{h_n}{f_6 \cdot f_7 \cdot \cdots \cdot f_n}$$

where $\mathbf{f_n} = (f_6, f_7, \ldots f_n)$ and f_k is the product of minors

$$f_k = (k-2\; k-1\; k)(k\; 1\; 2)(k-2\; 2\; 3)$$

- Six-points has only consecutive minors

$$A_6^{(3)} = \int_{\mathbf{f_6=0}} d\tau \frac{h_6(\tau)}{f_6(\tau)}$$

$$\mathbf{f_6} = (2)(4)(6) \qquad h_6 = \frac{1}{(1)(3)(5)}$$

Non-consecutive Minors: Seven-Point

$$A_7^{(3)} = \int_{\mathbf{F_7=0}} d^2\tau \frac{h_7}{f_6 \cdot f_7}$$

$$\mathbf{F_7} = (f_6, f_7) = ((2)(4)(612), (5)(7)(235)) \quad h_7 = \frac{(612)(235)}{(6)(1)(3)}$$

- Naively of 9 residues only $[(2)+(4)][(5)+(7)]$ contribute because (612) and (235) appear in the numerator
- $(2)=(235)=0$ means $2,3,4,5$ are on a line, so $(345)=0$; and this is the term $(2)(3)$. $(612)=(7)=0$ is $(6)(7)$
- This gives the BCFW contour

$$(2)[(3)+(5)+(7)]+(4)[(5)+(7)]+(6)(7)$$

Comments

- Unification of residues formula allows to think about n-point amplitude by "adding a particle" to the $(n-1)$-point amplitude in a way that makes the soft-limits manifest

$$\frac{h_n}{f_1 \cdot f_2 \cdots f_{M_n}} = \frac{h_{n-1}}{f_1 \cdot f_2 \cdots f_{M_{n-1}}} \times \mathcal{S}_{(n-1)\to n}$$

$$\mathcal{S}_{6\to 7} = \frac{1}{(671)} \times \frac{(561)(612)(235)}{f_7}$$

- Arkani-Hamed, Bourjaily, Cachazo, Trnka explicitly constructed single variety of NMHV and N^{2}MHV amplitudes

- How do you know the contour and single variety in general?
- Twistor string connected formula gives it to you

Seven points contour: from connected to ABCT

$$T_{7;3} = \int d^2\tau \frac{(235)(612)(135)(136)}{(1)(3)(6)} \frac{1}{S_1 S_2}$$

$$S_1 = (2)(4)(612)(135) - t_1(1)(3)(561)(246)$$

$$S_2 = (5)(7)(235)(136) - t_2(1)(3)(572)(671)$$

$$A_7^{(3)} = \int_{\mathbf{F_7=0}} d^2\tau \frac{h_7}{f_6 \cdot f_7}$$

$$\mathbf{F_7} = (f_6, f_7)$$

$$f_6 = (2)(4)(612) \quad f_7 = (5)(7)(235)$$

$$h_7 = \frac{(235)(612)}{(1)(3)(6)}$$

All Tree Formula

- The tree-level, planar, color-stripped, n-point N$^{(k-2)}$MHV amplitude can be written [Bourjaily, Trnka, AV, Wen]

$$A_n^{(k)} = \frac{1}{\text{vol}[\text{GL}(k)]} \int_{\Gamma_{n,k}(\text{F})} \frac{dC_{\alpha\, a} \;\; \text{H}_n^{(k)}}{(n-1)(1)(3)\;\, \text{F}_n^{(k)}} \prod_{\alpha=1}^{k} \delta^{4|4}\left(C_{\alpha\, a}\mathcal{W}_a\right)$$

where the contour is the zero-locus of $\text{F}_n^{(k)} \equiv \prod_{\ell=k+3}^{n}\left(\prod_{j=1}^{k-2} S_\ell^j\right)$

$S_\ell^j \equiv (1,\ldots,j-1,l+j-k,\ldots,l-3) \otimes S_{\ell-2\;\ell-1\;\ell:j\;\,j+1\;\,j+2}$

$$S_{\ell-2\;\ell-1\;\ell:j\;\,j+1\;\,j+2} = (\ell{-}2\;\ell{-}1\;\ell)\,(\ell\;j\;j{+}1)\,(j{+}1\;j{+}2\;\ell{-}2)\,(\ell{-}1\;j\;j{+}2) -$$
$$(j\;j{+}1\;j{+}2)\,(j{+}2\;\ell{-}2\;\ell{-}1)\,(\ell{-}1\;\ell\;j)\,(j{+}1\;\ell{-}2\;\ell)$$

Eight-Point Example: N^2MHV

- $n = 8 \quad k = 4 \quad j = 1,2 \quad l = 7,8$

$$A_8^{(4)} = \int_{\mathcal{S}_8} \frac{\mathcal{H}_8^4}{(1)(3)(7) \cdot \mathrm{F}_8^{(4)}}$$

$$\mathrm{F}_8^{(4)} = S_7^1 \cdot S_7^2 \cdot S_8^1 \cdot S_8^2$$

$$S_\ell^1 = (\ell - 3) \otimes S_{\ell-2\ \ell-1\ \ell:1\ 2\ 3} \qquad S_\ell^2 = (1) \otimes S_{\ell-2\ \ell-1\ \ell:2\ 3\ 4}$$

- Reproduces 20-term BCFW expression
- Note the difference from ABCT: formula is parity symmetric

Properties

- All tree formula can be obtained from the connected twistor string prescription via repeated application of the transformation

$$\delta(S_{ijk:rst})\delta(S_{ijk:rst'}) \sim \frac{c_{it}c_{jk:rt}}{c_{is}c_{jk:rs}}\delta(S_{ijk:rst})\delta(S_{ijk:rtt'})$$

- Parity symmetric for even number of particles
- Soft-limits are manifest
- Reproduces the correct YM amplitude

Summary

- RSV twistor string connected prescription in link variables is to be understood as the integrand of a contour integral

- Various different choices of contour compute various apparently different but actually equivalent representations for all tree amplitudes in SYM

- Simple deformation of the twistor string connected prescription integral gives Grassmanian integral written as the unified variety, thereby providing contour of integration

- The twistor string connected prescription is related to ACCK Grassmanian and BCFW expressions by a change of contour

- These statements are true for all tree amplitudes

Open Questions

- Loops?
- Gravity?
- Less SUSY?